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Non-equilibrium neutron stars

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Neutron stars contain superdense matter in their interiors. Characteristic densities in their cores are several times higher than the standard density of nuclear matter. This matter is so dense that it would be natural to assume that frequent particle collisions produce immediate equilibration. However, because of the slowness of some reactions, the equilibration with respect to them can be greatly delayed. Then one should deal with non-equilibrium stars which contain extra energy to be released. Deviations from equilibrium can affect neutrino emission of neutron stars, warm up their interiors and influence their thermal evolution. The effects of equilibration can be important for pulsating, rotating, accreting neutron stars, as well as for merging binary neutron stars.

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1. Introduction

Neutron stars (NSs) are known as the most dense and compact stellar objects; their masses are $M \sim 1.4 M_\odot$, and radii $R \sim 8-14$ km. It is currently accepted (e.g. Ref. 1) that NSs contain a thin crust (~ 1 km thick, $\sim 1\%$ by mass) and a massive core. Their mean mass densities $\overline{\rho} \sim (2-4)\rho_0$, and the central densities can exceed $10\ \rho_0$, where $\rho_0 \approx 2.8 \times 10^{14}$ g cm^{-3} is the standard density of nuclear matter. The outer crust ($\rho \lesssim 4 \times 10^{11}$ g cm^{-3}) consists mainly of strongly degenerate relativistic electrons (e) and fully ionized atomic nuclei which form Coulomb crystals. Deeper, in the inner crust (that extends to $\rho \sim 0.5\rho_0$), there are also free neutrons (n) dripped off the nuclei. At the bottom of the inner crust the nuclei may form clusters (nuclear pasta).[1] In the outer core, at $0.5\rho_0 \lesssim \rho \lesssim 2\rho_0$, the matter is a liquid of strongly degenerate neutrons, protons (p), electrons and muons, while deeper, in the inner core, other particles may appear such as hyperons, free quarks and the mixture of these. Nucleons, hyperons and quarks in NS interiors can be in superfluid state.[2] Despite many efforts, the composition and equation of state (EOS) of superdense matter in NS interiors are still poorly known, being the main challenge of NS physics.

As a rule, while constructing an EOS, one assumes full thermodynamic equilibrium with respect to all reaction channels. Indeed, particle collisions in a dense matter are so frequent that the equilibration should be almost instantaneous. For instance, a typical time between ordinary successive collisions in a NS core is $\tau_0 \sim (10^{-20} - 10^{-18})/T_8^2$ s (e.g. Ref. 3) which is a characteristic equilibration time among particles in a local matter element. Here T_8 is a core temperature T in units of 10^8 K.

However, there are important exclusions. Some reactions, mainly those associated with weak interaction, are so slow that the respective equilibration lasts for a long lime during which NSs stay out of full equilibrium. Such non-equilibrium NSs are outlined below. Naturally, their main tendency would be to equilibrate but the equilibration can occur in drastically different ways. The mechanisms which create and support non-equilibrium states are also discussed. The main ideas on non-equilibrium NSs were put forward long ago (e.g. Refs. 4, 5, 6, 7, 8, 9) but they have been temporarily forgotten and appeared to be useful later.

2. NS Cores: Three Non-Equlibrium Levels

There are many weak interaction processes in NS cores which mediate slow equilibration (e.g. Refs. 10, 11). Each process consists of two reactions (1 and 2), direct and inverse ones. These processes can be divided into three groups (Table 1).

Table 1. Three groups of weak equilibation processes.

Processes	Example (reactions 1 and 2)		τ_r
A) modified Urca	$nN \to peN\widetilde{\nu}_e$	$peN \to nN\nu_e$	~ 1 Myr$/T_8^6$
B) direct Urca	$n \to pe\widetilde{\nu}_e$	$pe \to n\nu_e$	~ 1 yr$/T_8^4$
C) non-leptonic	$n\Lambda \to p\Sigma^-$	$p\Sigma^- \to n\Lambda$	~ 1 s$/T_8^2$

The slowest are non-equilibrium modified Urca processes of group A first calculated in Ref. 12. For instance, they are important in the outer NS core composed of *npe*-matter. In this example, reaction 1 is just a neutron decay (which produces electron antineutrino $\widetilde{\nu}_e$), while reaction 2 is an electron capture on protons (produces neutrino ν_e). An additional nucleon N (n or p) is required to satisfy momentum conservation in a strongly degenerate matter of not too high density. Ordinary NSs are fully transparent to neutrinos. Neutrinos freely leave the star and cool it.

In full equilibrium, the rates Γ_1 and Γ_2 of the reactions 1 and 2 are naturally equal (being strong functions of T). In equilibrium, the numbers of n and p do not change with time, and neutrino and antineutrino emissivities are equal as well; the chemical potentials satisfy the familiar equality of 'chemical' equilibrium, $\mu_n = \mu_p + \mu_e$.

In the absence of equilibrium, this equality is violated. It is instructive to measure the strength of non-equilibration by the parameter $\delta\mu/kT$, where $\delta\mu = \mu_n - \mu_p - \mu_e$ is a disbalance of chemical potentials of reacting particles, and k is the Boltzmann constant. If $\delta\mu > 0$, one has an excess of neutrons. Then one gets $\Gamma_1 > \Gamma_2$ (emission of $\widetilde{\nu}_e$ is stronger than of ν_e) and the neutron excess will be reduced. If $\delta\mu < 0$, one has an excess of protons but $\Gamma_1 < \Gamma_2$ (with emission of ν_e being more intense) which again drives the system to equilibrium ($|\delta\mu| \to 0$).

One can introduce corresponding $\delta\mu$ for any equilibration process (Table 1). If the dense matter is composed of nucleons, group A contains the modified Urca processes involving electrons or muons with an additional nucleon, n or p. If the matter contains hyperons, the amount of processes A is much larger.[10]

Group B (first studied in Ref. 13) includes direct Urca processes which, as processes A, can be numerous but operate mainly in central parts of NS cores. In the simplest example of *npe*-matter they are just the reactions (1 and 2) of direct neutron decay and electron capture without additional nucleons. These processes are allowed in inner cores of NSs provided the proton fraction becomes sufficiently large.[14] Processes B are much faster than A but they are still slow by NS standards.

Group C contains non-leptonic processes which go through weak-interaction (strangeness changing) channels. They occur only in the matter containing strange particles, hyperons or free quarks. An example in Table 1 involves Λ and Σ^- hyperons. Another example is $nn \leftrightharpoons p\Sigma^-$. These processes are even faster than B (because they do not generate neutrinos) but, nevertheless, slow.

All non-equilibrium processes A–C are possible also in quark matter,[15–17] but we do not discuss them because of space restriction (see, e.g. Ref. 18 for bibliography).

In any case one can consider different deviations from equilibrium (e.g. Refs. 19, 10). The deviations with $|\delta\mu| \ll kT$ can be called sub-thermal. In this case $\Gamma_1 \approx \Gamma_2$, with $\delta\Gamma = \Gamma_1 - \Gamma_2 \approx -\lambda\delta\mu$, where λ is independent of $\delta\mu$. It is the so called linear regime. $\Gamma_{1,2}$ and λ can be calculated for a given model of matter. Larger deviations $|\delta\mu| \gg kT$ are supra-thermal and lead to very fast equilibration (decrease of $|\delta\mu|$). But once $|\delta\mu|$ becomes as low as $\sim kT$, the linear regime is established and the equilibration rate starts to be really slow. Therefore, it is the linear (or intermediate) regime which seems to be most important. It will be mainly considered below.

Having λ, one can estimate the relaxation time τ_r in the linear regime. As seen from Table 1, τ_r changes dramatically for different equilibration processes A–C.

3. NS Cores: Pulsations, Bulk Viscosity, Superfluidity

The relaxation is accompanied by excess energy release which is spent to NS heating (dissipation, a natural consequence of equilibration) and neutrino emission (for processes A and B). The relaxation is modified if a star pulsates and the pulsations (weak compressions/rarefactions of matter with typical frequencies $\omega \sim 10^4$ s^{-1}) penetrate into the core. Even if the matter was equilibrated in a non-pulsating NS, the pulsations regularly violate the equilibrium. Because of reactions (Table 1), the pulsations of particle densities and pressure are slightly shifted in phase. This converts the pulsation energy into the heat. In the linear pulsation regime such a process can be described in terms of effective bulk viscosity ζ that determines the dissipative heating rate Q_T [erg cm^{-3} s^{-1}] (e.g., Ref. 20).

In the simplest example of *npe*-matter, ζ can be written as

$$\zeta = \frac{C^2 n_b \tau_r}{B(1+\omega^2\tau_r^2)}, \quad C = n_b \frac{\partial \delta\mu}{\partial n_b}, \quad B = \left| \frac{\partial \delta\mu}{\partial X_p} \right|, \tag{1}$$

where n_b is the baryon number density, $\tau_r = n_b/(B\lambda)$ is the relaxation time, $X_p = n_p/n_b$ is the proton fraction (among baryons), while C and B are thermodynamic derivatives calculated for a given EOS neglecting pulsations.

This bulk viscosity depends on ω. For $\omega \sim 10^4$ s^{-1} one usually has $\omega\tau_r \gg 1$. In this high-frequency regime $\zeta \approx C^2 n_b/(B\omega^2\tau_r) \propto 1/(\omega^2\tau_r)$; it increases with decreasing ω and growing T. The associated viscous heating rate Q_T is independent of ω. In the strictly linear regime ($|\delta\mu| \ll kT$), Q_T remains smaller than the neutrino emissivity $Q_\nu \sim kT\Gamma$, but at $|\delta\mu| \gtrsim kT$ the heating rate Q_T exceeds Q_ν. These effects can influence the cooling of pulsating NSs.[4, 21]

Fig. 1. Bulk viscosity $\zeta \propto \omega^{-2}$ in a non-superfluid NS core pulsating at a frequency $\omega = 10^4$ s^{-1} as compared to the ordinary shear viscosity η (that is independent of ω, gray lines, formalism of Ref. 3). (a) Density dependence at three temperatures; (b) Temperature dependence at two baryon number densities n_b ($n_0 = 0.16$ fm^{-3} is the standard nuclear matter density).

Fig. 1 shows ζ as a function of n_b (a) and T (b) calculated at $\omega = 10^4$ s^{-1} for the same EOS of the *npe*μ matter (here μ labels muons) and in the same formalism as in Ref. 22. The low-density jump of ζ on Fig. 1(a) is due to the appearance of muons and the switch-on of muonic modified Urca process. The second (huge!) jump at $n_b \approx 4.5 n_0$ is due to the onset of electronic direct Urca (from A to B in Table 1), and the last jump is due to the muonic direct Urca. For comparison, gray lines show a standard shear viscosity $\eta \propto T^{-2}$ which is mediated by ordinary particle collisions and independent of ω. One can see that ζ dominates at sufficiently high T and ρ. Its behavior is much more sophisticated and drastically different from η.

At (unrealistic) very low pulsation frequencies ($\omega\tau_r \ll 1$) the bulk viscosity, Eq. (1), becomes static (ω-independent), $\zeta \approx C^2 n_b \tau_r/B$, and decreases with growing T. This static bulk viscosity is enormous, meaning that equilibration of quasi-static perturbations takes very long time, as discussed above.

The bulk viscosity in the matter of more complicated composition has similar features but is described by more cumbersome expressions.

So far we have neglected the effects of suprefluidity in NS cores. In the simplest models of nucleon cores, neutrons can be superfluid due to triplet-state pairing and protons due to singlet-state pairing.[2] Corresponding critical temperatures are expected to depend on density and reach $10^8 - 10^{10}$ K; they are very model dependent.

Other constituents of dense matter (hyperons, free quarks) can also be superfluid.

Superfluidity affects non-equilibrium processes in two ways.[10,20,23] Firstly, the presence of a gap in the energy spectra of particles (e.g., n or p) near respective Fermi-levels may strongly (exponentially) reduce the collision rates Γ which govern non-equilibrium processes. This will delay the equilibration. Secondly, superfluidity affects hydrodynamical motions (for instance, pulsations) in NS cores. Hydrodynamics becomes complicated;[23] it must describe motions of superfluid and normal components of matter. Instead of one bulk viscosity coefficient one should introduce several ones. The problem is further complicated if other particles like hyperons are present with their own superfluids. Other effects, which are not fully explored, are those of magnetic fields. Note that the hydrodynamic formalism should properly include the effects of General Relativity which are strong in NSs.

4. NS Crust

The matter in the crust can also be out of equilibrium for a long time. The most famous is an example of the accreted crust.[9] Even if the crust of a newly born NS is equilibrated (consists of the so called cold-catalyzed matter, e.g. Ref. 1) it can be driven out of equilibrium if the star enters a compact binary and accrets fresh material (mostly hydrogen and helium) from its companion. Eventually, the accreted matter will replace the initial crust.

Near the surface, light elements burn in nuclear reactions into iron-like nuclei. Subsequent evolution is determined by increasing compression under the weight of newly accreted matter (as if the accreted matter sinks into denser layers). The compression induces electron captures on nuclei, as well as emission and absorption of neutrons and, sometimes, pycnonuclear reactions.[9,24] At not too high temperatures available in the accreted crust, these transformations cannot convert the iron-like nuclei into cold-catalyzed ones because of huge Coulomb barriers. As a result, the accreted crust contains somewhat lighter nuclei and stays out of equilibrium until it is compressed to densities $\rho \sim 10^{13}$ g cm^{-3}. Only at higher ρ it merges finally into the cold-catalyzed matter. A typical energy excess in an accreted crust is $\sim 1-2$ MeV per one accreted nucleon. It is released mostly in the inner crust acting as the so called deep crustal heating mechanism.[25] The energy release is modest but occurs deeply in the star. It is sufficient to keep an accreting NS warm over timescales of $\sim$ Gyrs. In this case the equilibration goes not through weak interaction processes but through a chain of nuclear transformations. This allows one to heat the NS interior.

As mentioned in Sect. 1, at the bottom of the NS crust there could be a thin nuclear pasta layer. It can contain free neutrons and a small fraction of free protons moving in periodic (2D or 3D) exotic nuclear structures.[26] These neutrons and protons are actually quasi-free. In a periodic lattice, they acquire Bloch states, that modifies momentum conservation in their interactions. This opens an analogue of direct Urca process (with the neutrino emission and bulk viscosity) in the pasta although it appears to be weaker than in the core.[22,27]

Generally, modified and direct Urca processes on nuclei can operate everywhere in the NS crust. They are almost unexplored and most probably inefficient everywhere except in warm thin layers containing Urca-pairs of nuclei.[28–30]

5. Theory versus Observations

The damping of NS pulsations by non-equilibrium Ucra processes in NS cores has been widely discussed with regard to instabilities of rapidly rotating NSs driven by emission of gravitational waves (GWs). In particular, they are instabilities associated with Rossbi (r) modes. The bulk viscosity produces viscous damping of such instabilities, imposes serious constraints on the conditions at which the instabilities exist and can reheat a rapidly spinning NS (millisecond pulsar) keeping it sufficiently warm (which helps to explain observations), e.g. Refs. 31 and 32.

Non-equilibrium matter in NS cores can be created not only by NS pulsations, but also by various slow deformations of the core configuration which drive the matter out of equilibrium. For instance, this may happen if a spinning NS (a pulsar) slows down via magnetic braking (magnetic dipole radiation mechanism). The spindown makes the star more spherical producing deviations from weak-interaction equilibrium. The deviations trigger a slow relaxation and heating of the NS core, and consequently, the entire star. This is a valid 'roto-chemical' heating mechanism which is used to explain unexpectedly not too low temperature of the old millisecond pulsar J0437–4715 (see Ref. 33 for bibliography). Similar heating can be produced by deformations of deep NS crust under magnetic braking.[34] New observations of old but not too cold (currently non-accreting) NSs are required to elaborate their reheating mechanisms and to reliably infer important information on internal structure of NSs.

Many applications are done for transiently accreting NSs in low-mass X-ray binaries. From time to time these NSs undergo accretion episodes during which they are observed as bright X-ray sources powered by the huge release of free-fall energy of the accreted matter at the NS surface. This energy does not heat the star but is mostly emitted from the NS surface. Periods of accretion are superimposed by longer periods of quiescence. Such NSs are old; they should have been cold inside and be not observable in quiescence. Nevertheless, they are shining in X-rays during quiescent epochs, indicating that they are warmed up (see Refs. 35, 36 for bibliography). Their warmness is mostly explained by the deep crustal heating mechanism.[24, 25]

6. New Era: GW Astronomy and Binary NS Mergers

The advent of practical GW astronomy and the first observation of the GW 170817 event of binary NS merging opened a new era in NS physics.[37] When two NSs merge, the tidal forces become very strong. The structure of merging NSs varies over dynamical NS timescales ~ 1 ms and the variations are strong to destroy the entire NS configuration. Under such conditions, NS matter can be heated to temperatures

$kT \sim 5-50$ MeV or higher. The EOS of NS matter becomes essentially temperature dependent, the matter can be partly or fully opaque to neutrinos. Hydrodynamic motions and pulsations can be excited there. Needless to say, this matter can be out of equilibrium. This would inevitably cause strong dissipative processes which would greatly affect the merging dynamics. The role of non-equilibrium and associated dissipation in merging NSs can be curcial for a correct interpretation of GW and multi-wave electromagnetic observations. The work in this line has already been started (see, e.g., Refs. 38, 39 for bibliography) but many new interesting problems have to be solved to understand the details of merging phenomena.

7. Conclusion

Although NSs are very dense inside, their matter can be out of equilibrium with respect to some slow reactions. Non-equilibrium states may have different origin and occur in a NS core or a crust. They can be supported by NS pulsations, by accretion, by various NS deformations (e.g. due to magnetic braking) and manifest themselves in a variety of observable phenomena. For instance, they can support old and warm millisecond pulsars and transiently accreting NSs in low-mass X-ray binaries. Non-equilibrium processes should play crucial role in merging NSs observed by GW observatories. This is a new excited field to be explored.

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