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Magnetism of massive stars in the early Universe

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ABSTRACT

It is generally believed that the first stars were hot and massive because of the lack of efficient coolants in the metal-free primordial gas. This paper considers the thermal generation of the magnetic field in such stars. The mechanism operates in the surface layers of hot stars where departures from the local thermodynamic equilibrium form a region with the inverse temperature gradient (it occurs in atmospheric layers with the optical depth $\leq$0.01–0.001). The thermal generation is efficient in stars with a low mass-loss rate. A growth rate of the magnetic field can be such high that even young stars with the age $\sim 10^4$–10^5 yr possess the magnetic field comparable to that detected in massive stars of the present-day Universe.

Key words: stars: atmospheres – stars: magnetic field – stars: massive.

1 INTRODUCTION

The first generation of stars (Population III) plays an important role in the early Universe. For instance, they determine the chemical evolution of the Universe because elements heavier than helium are produced by nuclear reactions in massive stars. The interstellar medium is enriched by such elements from massive stars via supernova explosions and stellar winds. Since the primordial gas does not contain efficient coolants it is generally believed that the first stars formed from such gas were more massive. The typical mass of the first stars may reach 100–300 $M_{\odot}$ (Yoon, Dierks & Langer 2012). Their initial radius and surface temperature are of the order of 3–20 $R_{\odot}$ and 40 000–60 000 K, respectively; $R_{\odot}$ is the solar radius (see e.g. Krtićka & Kubat 2006b). Such stars can also be important sources of ultraviolet photons for reionization. Some of the first stars may produce very bright events such as supernovae or gamma-ray bursts, which are potentially observable. The abundances observed in early massive stars at high redshift might provide information on the nucleosyntheses in the early Universe (Cayrel et al. 2004; Erni et al. 2006; Kobayashi, Tominaga & Nomoto 2011).

One of the important questions concerning these stars is how the hydrodynamic processes can influence the properties and evolution of first stars. Generally, both the rotationally induced chemical mixing and magnetohydrodynamic (MHD) motions, caused by a generated magnetic field, can alter the mass-loss rate and the stellar structure of metal-poor massive stars (see e.g. Heger & Woosley 2010; Yoon et al. 2012). Perhaps, the influence of the magnetic field can be particularly important in such stars because the angular momentum loss is usually inefficient in stars with stellar wind driven by radiation (Maeder & Meynet 2000).

Unfortunately, there is no consensus regarding the origin and topology of the magnetic field in massive stars. As a result, there is some problem to predict the influence of the magnetic field on properties and evolution of the first stars. Very few studies considered these problems. For instance, Yoon et al. (2012) analysed the influence of rotation and magnetic field on metal-poor stars. These authors assumed that the magnetic torques is caused by the Spruit–Tayler dynamo (Spruit 2002) and neglected any other possibilities. Unfortunately, there are no pieces of evidence that the Spruit–Tayler dynamo operates in massive stars. The evolution and structure of stars may differ qualitatively if the magnetic field is generated by other mechanism and its topology differs from that described by Spruit (2002).

Recently, the magnetic field has been detected in a number of massive B- and O-type stars (see e.g. Hubrig et al. 2016; Schöller et al. 2017. Basically, the magnetic fields of massive present-day stars are several times weaker than the well-studied fields of A stars , but even such relatively weak fields can be important for magnetohydrodynamic processes in massive stars and influence the evolution of their environment.

The mechanism generating magnetic fields in massive stars is still a subject of debate. Present-day massive stars have no convective zones and, likely, their magnetic field cannot be generated by convective dynamo. Some authors assume that convective zone in massive stars occurs because of ionization of *Fe* in subsurface layers (see e.g. Cantiello & Braithwaite 2011). This version of dynamo cannot operate in Population III stars, however, since elements heavier than *He* are provided in the Early Universe by massive stars themselves. Apart from the convective dynamo, some other mechanisms can be responsible for a field generation as well. For instance, the well-known example of such mechanism is the Biermann battery (see e.g. Biermann 1950, Kemp 1982; Mestel & Moss 1983) that can operate even in the case of vanishing hydrodynamic velocity. Note that this mechanism in plasma is similar to the so-

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called cross-helicity effect that can operate in a conducting turbulent fluid under certain conditions (e.g., Brandenburg & Urpin 1998).

The considered mechanism is based on the Nernst effect and does not require hydrodynamic motions. This mechanism was studied first regarding experiments with the laser-heated plasma (see e.g. Tidman & Shanny 1974; Dolginov & Urpin 1979; Haines 1981; Andrushchenko & Pavlenko 2004; Bissell, Kingham & Ridgers 2012; Bissell 2015). Under astrophysical conditions, this mechanism has been proposed to account for the origin of a strong magnetic field in the solid crust of neutron stars by Blandford, Applegate & Hernquist (1983); later, Urpin, Levshakov & Yakovlev (1986) (see also Geppert & Wiebicke 1991) have shown that the thermomagnetic generation can operate in liquid layers of neutron stars as well. The thermally generated magnetic fields explain some qualitative features in the magnetic evolution of neutron stars (see Urpin & van Riper 1993; Urpin, Chanmugam & Sang 1994).

In this paper, we consider a generation of the magnetic field by this mechanism in massive Population III stars. It was shown in the previous paper that the thermomagnetic generation can explain the origin of the magnetic field in present-day massive stars (Urpin 2017).

In this paper, we consider in details the influence of the stellar wind on a field generation. Plasma in the surface layers of fist stars is hot and, therefore, the thermomagnetic processes can operate efficiently there. However, it will be shown that mass-loss can prevent a field amplification in stars with a high mass-loss rate. Therefore, generation is efficient only in massive stars with a low mass-loss rate.

2 BASIC EQUATIONS

We consider a generation of the magnetic field in the surface region of massive stars. Since the rate of thermomagnetic processes decreases rapidly with a depth from the surface, we can assume that the thickness of a layer of the efficient generation is small compared to the stellar radius, R. We mimic this region as a slab between $z = 0$ (bottom) and $z = a$ (surface) ($R \gg a$). We use the Cartesian coordinates (x, y, z) and assume that the temperature T depends on the z-coordinate alone. In fully ionized plasma with $T = T(z)$, the Ohm's law can be represented as

$$\vec{E} = -\frac{\vec{v}}{c} \times \vec{B} - \frac{\vec{B} \times \vec{j}}{en_{\rm e}} - \frac{\nabla p_{\rm e}}{en_{\rm e}} + \frac{\hat{\alpha} \cdot \vec{j}}{(en_{\rm e})^2} - \frac{\hat{\beta} \cdot \nabla T}{en_{\rm e}}, \quad (1)$$

where

$$\hat{\alpha} \cdot \vec{j} = \alpha_{\parallel} j_{\parallel} + \alpha_{\perp} j_{\perp} - \alpha_{\wedge} \vec{b} \times \vec{j}, \quad (2)$$

$$\hat{\beta} \cdot \nabla T = \beta_{\parallel} \nabla_{\parallel} T + \beta_{\perp} \nabla_{\perp} T + \beta_{\wedge} \vec{b} \times \nabla T, \quad (3)$$

(see Braginskii 1965). Here $\vec{E}$, $\vec{B}$, and $\vec{j}$ are the electric and magnetic fields and the electric current, respectively; $\vec{v}$ is the hydrodynamic velocity, $n_{\rm e}$ and $p_{\rm e}$ are the number density and pressure of electrons; e is the electron charge. The coefficients α and β have been calculated by Braginskii (1965) and represent the Hall effect and the Nernst effect, respectively; the subscripts $\parallel$, $\perp$, and $\wedge$ mark components parallel and perpendicular to the magnetic field and the Hall component; $\vec{b} = \vec{B}/B$.

Generation of the magnetic field by the Nernst effect is relevant to the excitation of thermomagnetic modes (Tidman & Shanny 1974; Dolginov & Urpin 1979). The induction equation governing the magnetic evolution can be obtained by combining equation (1) with the Faraday's law. We consider the stability of thermomagnetic modes and accompanying generation of the magnetic field by making use of a linear approach. In this approach, all quantities can be represented as a sum of the unperturbed quantity and a small perturbation that will be marked in this paper by a subscript 1. Perturbations are governed by linearized equations. In the unperturbed state, we assume that $\nabla T \neq 0$ and $B = 0$. The linearized induction equation can be represented as (see e.g. Urpin 2017)

$$\frac{\partial \vec{B}_1}{\partial t} = \nabla \times (\vec{v}_0 \times \vec{B}_1) - \nabla \times \left(\eta_{\rm m} \nabla \times \vec{B}_1 \right) +$$
$$\frac{c}{e} \nabla \times \left(\frac{\nabla p_{\rm e1}}{n_{\rm e}} - \frac{n_{\rm e1} \nabla p_{\rm e}}{n_{\rm e}^2} \right) - 0.81 \frac{k_{\rm B}}{m_{\rm e}} \nabla \times (\tau_{\rm e} \nabla T \times \vec{B}_1), \quad (4)$$

where k_B is the Boltzmann constant, $\tau_{\rm e} = 3\sqrt{m_{\rm e}}(k_{\rm B}T)^{3/2}/4\sqrt{2\pi} e^4 n_{\rm e} \Lambda$ is the electron relaxation time (see e.g. Braginskii 1965; Spitzer 1998), Λ is the Coulomb logarithm; $\eta_{\rm m} = c^2/4\pi\sigma$ is the magnetic diffusivity, and $\sigma = e^2 n_{\rm e} \tau_{\rm e}/0.51 m_{\rm e}$ is the conductivity along the magnetic field. The linearized kinetic coefficients in equation (4) have been calculated for a hydrogen plasma by Urpin (2017). The main difference to the induction equation considered by Urpin (2017) is the presence of the first term on the r.h.s of equation (4). This term describes the effect of a flow caused by mass-loss due to a stellar wind. Such flow can play an essential role in massive stars where the mass-loss rate is high. We assume that the only hydrodynamic motion is caused by a stellar wind and the velocity of this flow is directed in a positive z-direction. If the mass-loss rate is $\dot{M}$, then this velocity is given by

$$v_0 = \dot{M}/4\pi R^2 \rho, \quad (5)$$

where ρ is the density. Since ρ depends on z, the velocity v_0 is also dependent on z in our model.

Induction equation (4) is sufficient to describe the thermal generation of the magnetic field only in some particular cases (Urpin 2017). Generally, this equation should be complemented by the heat balance, momentum, and continuity equations. The momentum and continuity equations can be represented as

$$\frac{{\rm d}\vec{v}}{{\rm d}t} = -\frac{\nabla p}{\rho} + \vec{g} + \frac{1}{4\pi} (\nabla \times \vec{B}) \times \vec{B}, \quad (6)$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0, \quad (7)$$

where $\vec{g}$ is gravity. Note that linearization of equation (6) and (7) does not contain linear terms in $\vec{B}_1$ because the magnetic field is vanishing in the basic state. Linearization of the momentum and continuity yields only linear equations containing T_1, ρ_1, p_1, and $\vec{v}_1$.

The heat balance is governed by

$$\rho c_{\rm p} \frac{{\rm d}T}{{\rm d}t} - \frac{{\rm d}p}{{\rm d}t} = -\nabla \cdot \vec{q}_{\rm e} - \nabla \cdot \vec{Q} + G - \Lambda, \quad (8)$$

where p and $c_{\rm p}$ are the pressure and specific heat of plasma for $p =$ const, respectively; G and R are the heating and cooling rates; $\vec{q}_{\rm e} = -\hat{\kappa}_{\rm e} \cdot \nabla T$ is the electron heat flux with $\hat{\kappa}_{\rm e}$ being the tensor of electron thermal conductivity; $\vec{Q}$ is the radiative heat flux; ${\rm d}/{\rm d}t = \partial/\partial t + (\vec{v} \cdot \nabla)$. The tensor of electron thermal conductivity, $\hat{\kappa}_{\rm e}$, has a standard form (see e.g. Braginskii 1965). If the magnetic field is not very strong (see discussion below), the radiative heat flux, $\vec{Q}$, has a very simple form in optically thick layers, $\vec{Q} = -\kappa_{\rm r} \nabla T$, where $\kappa_{\rm r}$ is the radiation thermal conductivity (see Schwarzschild 1958). However, the expression for a radiative flux is rather complex in the region with a small optical depth, ≤1. In our calculations, we do not need the expression for $\vec{Q}$ with a high accuracy and, therefore, we can use one of the approximate models. Various approximations have been suggested to describe the quantity $\nabla \cdot \vec{Q}$ in the region with a small optical depth. For our purposes, for instance, we can use the approach suggested by Wang (1966). According to this approach, $\nabla \cdot \vec{Q}$ can be represented as a power-law function of ρ and T,

$$\nabla \cdot \vec{Q} \approx acW_0 \rho^{1+f_1} T^{4+f_2}, \tag{9}$$

where a is the Stefan–Boltzmann constant and W_0, f_1, and f_2 are constants determined by fitting with opacity data.

Our results on the field generation do not depend on a particular form of equation (9). The only important point in this equation is that the quantity $\nabla \cdot \vec{Q}$ do not depend on the magnetic field, B. But this is exactly the case of massive stars. The absorption coefficients in hot stars begin to be influenced by B if the magnetic field is sufficiently strong, $B \sim 10^8$–10^9 G. In hot massive stars, the magnetic field is of the order of 10^2–10^3 G and such a field does not influence radiation transport coefficients.

Usually, in massive stars, the electron heat transport is several orders of magnitude weaker, at least, than the radiative transport (see discussion in Urpin 2017). Therefore, the term with $q_{\rm e}$ in equation (8) can be neglected. Linearization of equations (4)–(9) yields in the general case to the set of equations containing all disturbances. If $q_{\rm e}$ is negligible then this set does not contain $\vec{B}_1$. This occurs because, if $q_{\rm e}$ is neglected, the only terms in equations (4) and (6)–(8), containing $\vec{B}$, are non-linear terms. Linearization of these terms produces a vanishing contribution since $\vec{B} = 0$ and $\nabla \times \vec{B} = 0$ in the basic state. Therefore, linearized equation (4) is decoupled with equations (6)–(8) and forms its own set of eigenmodes. These eigenmodes are related to the Nernst effect and determined by the perturbations of $\vec{B}_1$ alone. All perturbations of other quantities are vanishing.

The equation governing these particular modes has a shape

$$\frac{\partial \vec{B}_1}{\partial t} = \nabla \times (v_0 \vec{e}_z \times \vec{B}_1) - \nabla \times \Big[\frac{k_{\rm B}\tau_{\rm e}}{m_{\rm e}}\Big(0.81\frac{\nabla T}{T} \times \vec{B}_1\Big) + \frac{0,51}{\varepsilon}(\nabla \times \vec{B}_1)\Big] \quad , \quad \varepsilon \equiv \frac{c_{\rm e}^2}{c^2}\,\omega_{\rm p}^2\tau_{\rm e}^2 \gg 1, \tag{10}$$

where $c_{\rm e} = \sqrt{k_{\rm B}T/m_{\rm e}}$ is the thermal velocity of electrons and $\omega_{\rm p} = \sqrt{4\pi e^2 n_{\rm e}/m_{\rm e}}$ the plasma frequency. The second and third terms in the square brackets on the r.h.s. of equation (10) describe thermomagnetic and dissipative effects, respectively. Comparing these terms and assuming that the length-scales of perturbations and unperturbed quantities are of the same order, we obtain that thermomagnetic effects yield a stronger influence than Ohmic dissipation if $\varepsilon \equiv \frac{c_{\rm e}^2}{c^2}\,\omega_{\rm p}^2\tau_{\rm e}^2 \gg 1$.

3 MAGNETIC FIELD GENERATION

In our model, $\vec{B}_1$ is perpendicular to ∇T and located in the plane (x, y). We choose $\vec{B}_1$ being parallel to the y-axis. The basic state is assumed to be quasi-stationary and uniform in the x-direction. Then, $\vec{B}_1$ is proportional to $\exp(\gamma t - ik_x x)$ with γ being the frequency/(growth rate) and k_x the wavevector in the x-direction. Then the equation for B_{1y} can be written in the form

$$\eta_m B_{1y}'' + AB_{1y}' + DB_{1y} = 0, \tag{11}$$

where

$$A = 0.81\frac{k_{\rm B}}{m_{\rm e}}\tau_{\rm e}\frac{{\rm d}T}{{\rm d}z} - \eta_{\rm m}\frac{{\rm d}\ln\sigma}{{\rm d}z} - v_0,$$
$$D = D_0 - \eta_{\rm m}k_x^2 - v_0' - \gamma,$$
$$D_0 = 0.81\frac{k_{\rm B}}{m_{\rm e}}\frac{\rm d}{{\rm d}z}\left(\tau_{\rm e}\frac{{\rm d}T}{{\rm d}z}\right); \tag{12}$$

the prime denotes d/dz.

Consider the instability of thermomagnetic modes assuming that a wavelength is short in the z-direction. In this case, we can use the WKB-approximation and represent the solution of equation (11) as

$$B_{1y} = F_1 \exp\left[i\int q_1(z){\rm d}z\right] + F_2 \exp\left[i\int q_2(z){\rm d}z\right], \tag{13}$$

where F_1 and F_2 are constants that are determined by boundary conditions. Functions $q_{1,\,2}$ are the vertical wavevectors that can be calculated from equation (11)

$$q^2 - \frac{iA}{\eta_{\rm m}}q - \frac{D}{\eta_{\rm m}} - iq' = 0. \tag{14}$$

The last term on the l.h.s. of this equation is small compared to other terms for short wavelength perturbations and therefore one can calculate q as a series of subsequent perturbation terms, $q = q^{(0)} + q^{(1)}$.... Restricting ourselves in first two terms, we obtain

$$q_{1,2}^{(0)} = \frac{iA}{2\eta_{\rm m}} \pm \sqrt{\frac{D}{\eta_{\rm m}} - \frac{A^2}{4\eta_{\rm m}^2}},\; q^{(1)} = \frac{iq'^{(0)}}{2q^{(0)} - iA/\eta_{\rm m}}. \tag{15}$$

As it was noted, the ratio between F_1 and F_2 should be determined from the boundary conditions. We consider the simplest possible example of boundary conditions. We assume that the magnetic field vanishes at the bottom of the generating region ($B_y = 0$ at $z = 0$) because the thermomagnetic effects become too slow in deep layers and cannot provide an efficient generation there. The second boundary condition can be taken from the following reasons. The thermal mechanism generates only the toroidal magnetic field, B_y, but the toroidal field should vanish outside the star and, hence, the electric current is equal to zero at the stellar surface (dB_y/d$z = 0$ at $z = a$).

Then, we have $F_1 = -F_2$ from the first boundary condition and, hence, the solution of equation (13) takes the shape

$$B_{1y} = F_1\left\{\exp\left[i\int_0^z q_1(z){\rm d}z\right] - \exp\left[i\int_0^z q_2(z){\rm d}z\right]\right\}. \tag{16}$$

The boundary condition at the stellar surface yields

$$q_1(a) - q_2(a)\exp\left[i\int_0^a [q_2(z) - q_1(z)]{\rm d}z\right] = 0. \tag{17}$$

By making use of equation (15) for $q_{1,\,2}$, we have

$$q_1(a) - q_2(a)\exp\left[-2i\int_0^a \sqrt{\frac{D}{\eta_{\rm m}} - \frac{A^2}{4\eta_{\rm m}^2}}{\rm d}z\right] = 0. \tag{18}$$

From equation (12), one can estimate A and D as $A \sim c_{\rm e}^2\tau_{\rm e}/L$ and $D \sim c_{\rm e}^2\tau_{\rm e}/L^2$. Then we obtain $(D/\eta_{\rm m})/(A^2/4\eta_{\rm m}^2) \ll 1$. The square root in this equation can be represented as

$$\sqrt{\frac{D}{\eta_{\rm m}} - \frac{A^2}{4\eta_{\rm m}^2}} \approx \frac{iA}{2\eta_{\rm m}}\left(1 - \frac{2\eta_{\rm m}D}{A^2}\right) \tag{19}$$

and therefore the second boundary condition takes the form

$$q_1(a) - q_2(a)\exp\left[\int_0^a (A/\eta_{\rm m}){\rm d}z\right] = 0. \tag{20}$$

The solution of this equation depends crucially on the sign of A. In the region, where thermomagnetic effects play a dominating role, this sign is determined by $\mathrm{d}T/\mathrm{d}z$). If $\mathrm{d}T/\mathrm{d}z < 0$ and the temperature decreases outward then the exponential term on the l.h.s. of equation (20) is small. Therefore, one can neglect this term. Then, the dispersion relation reads in this case $q_1(a) \approx 0$. Substituting equation (15) into this dispersion relation, we have

$$q_1^{(0)} \approx \frac{iA}{\eta_\mathrm{m}} - \frac{iD}{A}, \quad q_2^{(0)} \approx \frac{iD}{A}. \tag{21}$$

The condition $q_1^{(o)}(a) \approx 0$ yields

$$\frac{A}{\eta_\mathrm{m}} - \frac{D}{A} \approx 0 \tag{22}$$

or, using equation (12),

$$\gamma \approx -\frac{A^2}{\eta_\mathrm{m}} - v_0' + D_0. \tag{23}$$

Since $A^2/\eta_\mathrm{m} \gg D_0$ in our model, the expression for γ can be transformed into $\gamma \approx -A^2/\eta_\mathrm{m} - v_0'$. The first term on the r.h.s. of this equation gives a negative contribution to γ. The quantity $v_0 \propto \rho^{-1}$ increases with r in the atmosphere and therefore the contribution of the second term is also negative. Hence, generation of the magnetic field by thermomagnetic effects is impossible if the temperature decreases outward and $\mathrm{d}T/\mathrm{d}z < 0$.

In the region with the inverse temperature gradient, $\mathrm{d}T/\mathrm{d}z > 0$, a behaviour of eigenroots is qualitatively different. In this region, the second term on the l.h.s. of equation (20) gives the main contribution to this equation since $\int_0^a (A/\eta_\mathrm{m})\mathrm{d}z \gg 1$. Therefore, one can neglect the first term on the l.h.s. Then, the dispersion relation takes the form $q_2(a) \approx 0$ or $D(a) = 0$. It can be written as

$$\gamma \approx D_0(a) - v_0' \approx 0.81 \frac{k_\mathrm{B}}{m_\mathrm{e}} \frac{\mathrm{d}}{\mathrm{d}z}\left(\tau_\mathrm{e}\frac{\mathrm{d}T}{\mathrm{d}z}\right) - v_0'. \tag{24}$$

In fully ionized plasma, $\tau_\mathrm{e} \propto T^{3/2}/n$ (see e.g. Braginskii 1965; Spitzer 1998) and, hence,

$$\gamma \sim 0.81 \frac{k_\mathrm{B}}{m_\mathrm{e}} \tau_\mathrm{e} \left[\frac{3}{2T}\left(\frac{\mathrm{d}T}{\mathrm{d}z}\right)^2 - \frac{\mathrm{d}\ln\rho}{\mathrm{d}z}\frac{\mathrm{d}T}{\mathrm{d}z} + \frac{\mathrm{d}^2T}{\mathrm{d}z^2}\right] - v_0'. \tag{25}$$

If the mass-loss rate is low, the growth rate of instability has been considered by Urpin (2017). In this case, the last term on the r.h.s. of equation (24) is small and one can neglect it. It turns that for a low mass-loss rate, the sign of γ depends crucially on the temperature profile. The first term on the r.h.s. of equation (24) describes the influence of thermomagnetic effects on the field generation/dissipation, and the last term on the r.h.s. originates from advection of the field lines by a stellar wind. A plasma flow carries out the field lines from the region where they are generated and, as a result, such advection decreases the generated field. Since these two effects are of the opposite sign, resulting γ can be positive in some atmospheric layers. It appears that γ can be positive in layers with the inverse temperature gradient (Urpin 2017). Such layers certainly are formed in massive stars because of their high luminosity. The temperature profile in massive stars is well studied since the papers by Auer & Mihalas (1969a, b). These pioneering calculations have been confirmed later by many authors (see e.g. Gabler et al. 1989 and Martins 2004 for review). Calculations show that the temperature profile in massive stars has a bump-like structure. The bump is usually located in the region with a small optical depth, ∼0.01–0.001 (see e.g. fig. 2.1 of Martins 2004). In our simplified model, we can assume that the bottom of a generating layer, $z = 0$, corresponds approximately to the depth where the temperature gradient changes the sign since the field generation can occur only in a region with the inverse temperature gradient. Then, it is easy to check that $\gamma > 0$ in a fraction of the bump region, at least (see Urpin 2017). Therefore, there always exists the region where the instability occurs. The typical high of this region is comparable with the thickness of a layer with the inverse temperature gradient.

Layers with the inverse temperature gradient are formed in a wide variety of massive stars. They exist in stars with essentially different masses and chemical compositions (see e.g. Auer & Mihalas 1969a, b; Gabler et al. 1989, and Martins 2004). The main reason why such layers occur is a high luminosity of massive stars. Therefore one can expect that the regions with the inverse temperature gradient exist in the first stars and the considered mechanism of the magnetic field generation operate in Population III stars if the mass-loss rate is not very high.

4 DISCUSSION

This paper considers the thermal mechanism of a magnetic field generation in Population III stars. The mechanism is based on the Nernst effect and differs qualitatively from the standard dynamo because it does not require hydrodynamic motions. The thermal generation can occur in the atmosphere of hot stars because there exist a region with the inverse temperature gradient. It is generally believed that the first stars were massive and hot because of the lack of efficient coolants in the metal-free primordial gas. The occurence of the inverse temperature gradient is well known from the atmospheric modelling of massive stars and is a rather general feature of such stars (see e.g. Auer & Mihalas 1969a, b; Martins 2004). The inverse temperature gradient usually exists in the region with a small optical depth, $\tau \sim 0.01$–0.001 (see e.g. Martins 2004). Note that such a generation region may occur in massive stars with different masses and chemical compositions. The atmospheric models of massive stars show that the layer with the inverse temperature gradient can be formed even in plasma with the composition similar to the primordial gas (pure H or a mixture $H + He$; see e.g. Auer & Mihalas 1969a, b; Martins 2004). Likely, regions with the inverse temperature gradient are formed in the first stars as well and these regions can be responsible for the magnetic fields of such stars. The Population III stars play a key role in the early Universe since they influence a local environment via energy deposition. The chemical evolution of the early Universe also is determined by massive stars because elements heavier than He are basically provided by these stars via supernova explosions and stellar winds. Likely, the magnetic field generated by the thermal mechanism can influence some of these processes.

The field generation time, $t_B = 1/\gamma$, is rather short and often is shorter than (or comparable to) the lifetime of massive stars. Therefore, these stars have generally enough time to amplify the magnetic field. Using equation (25), a growth rate of the magnetic field can be estimated as

$$\gamma \sim (k_\mathrm{B}/m_\mathrm{e})\tau_\mathrm{e}(3/2T)(\mathrm{d}T/\mathrm{d}z))^2 - v_0' \sim c_\mathrm{e}(\lambda_\mathrm{e}/L^2) - v_0', \tag{26}$$

where $\lambda_\mathrm{e} = c_\mathrm{e}\tau_\mathrm{e}$ is the mean-free path of electrons. Using the standard expression for the electron relaxation time (see Spitzer 1998), we obtain the following estimate of the growth time for stars with low mass-loss rate

$$t_B \sim 10^3\, n_{13}\, L_9^2\, \Lambda T_4^{-5/2}\mathrm{yr}, \tag{27}$$

where $L_9 = L/10^9$ cm. For typical atmospheric parameters of the early massive stars ($n_{13} = L_9 = 1$, $\Lambda = 4$, $T_4 = 5$), we have $t_B \sim 10^4$–10^5 yr if $L_9 \sim 1$.

The lifetime of massive stars, $\tau_{\rm ms}$, is of the order of

$$\tau_{\rm ms} \approx 10^{10}(M/M_{\odot})^{-2.5}\text{yr}, \tag{28}$$

where M is the stellar mass (see e.g. Bhattacharya & van den Heuvel 1991; Urpin, Konenkov & Geppert 1998). Therefore, the generation time is shorter than the evolution time-scale for stars with $M \leq 100$–$300\,{\rm M}_{\odot}$ and the field generation can occur in such stars. However, the considered mechanism cannot explain a generation of the magnetic fields in very massive stars because their lifetime is shorter than t_B. Therefore, the thermal mechanism is likely inefficient in some massive stars of the Population III. Also, the considered mechanism cannot provide the magnetic field in very young stars (with the age $<10^4$ yr). Nevertheless, Population III stars with a smaller mass ($M \leq 100$–$300\,{\rm M}_{\odot}$) can possess the magnetic field of the same order of magnitude as currently observed massive stars.

One more important effect that influences the magnetic field in massive stars is the stellar wind. Advection of the field lines by a wind decreases substantially generation rate if the mass-loss rate is high. A contribution of the wind to the magnetic field evolution is given by the last term on the r.h.s. of equation (25). This term is negative since $v_0 > 0$ and can be estimated as V_w/L_w where V_w is the wind velocity. Then, the characteristic time-scale of advection is $\sim L_w/V_w$. The order of magnitude of this velocity is $V_w \sim \dot{M}/4\pi R^2 \rho$ (see equation 5) where $\dot{M}$ is the rate of mass-loss and R is the stellar radius; $\rho = m_{\rm p} n$. Then, the advective time-scale is $t_W \approx 4\pi R^2 \rho (L_w/\dot{M})$. The condition that the influence of a stellar wind on the magnetic field is less important than the thermal generation reads $\gamma > 0$ or

$$\dot{M} < 2 \times 10^{-14} \left(\frac{R_{11}}{L_9} \right)^2 \frac{T_4^{5/2}}{\Lambda_{10}} L_{w11} \tag{29}$$

where $\dot{M}$ is in units ${\rm M}_{\odot}\,{\rm yr}^{-1}$ and $R_{11} = R/10^{11}$ cm, $L_{w11} = L_w/10^{11}$ cm.

Obviously, a very high mass-loss rate leads to negative γ and prevents the magnetic field generation. Even in very deep layers where the temperature is $\sim 10^7$K or higher, the condition (29) can be satisfied only if the rate of mass-loss is sufficiently low, $\dot{M} < 10^{-8}\,{\rm M}_{\odot}\,{\rm yr}^{-1}$. Near the surface, the thermal generation becomes possible if the mass-loss rate is lower. For instance, in sub-photospheric layers with $T \sim 3 \times 10^5$K, the thermomagnetic instability is efficient if $\dot{M} < 10^{-12}\,{\rm M}_{\odot}\,{\rm yr}^{-1}$. The rate of mass-loss is typically relatively high in massive stars (see e.g. Lamers 1981) and, therefore, the condition (29) is rather difficult to satisfy in present-day massive stars. As a result, the number of magnetic massive stars is relatively small at present ($<$ 10 per cent). However, the mass-loss rate can be essentially lower in Population III stars. The very first stars were formed completely metal-free. Present-day hot stars have radiatively driven winds due to transitions of elements heavier than *H* and *He* (like carbon, nitrogen, oxygen, or iron). The radiative force is essentially weaker in stars formed from the primordial gas. Therefore, the radiative force is not able to drive a significant stellar wind from hot massive zero-metallicity stars (Krtićka & Kubat 2006a, b; Muijres et al. 2012). For instance, numerical calculations by Krtićka & Kubat (2006a) show that the mass-loss rate is about $\dot{M} \approx 10^{-14}$–$10^{-16}\,{\rm M}_{\odot}\,{\rm yr}^{-1}$ for such stars.

Note that evolutionary tracks of Population III stars show some surface enrichment caused by mixing of CNO-processed material. This enrichment can generally enhance the mass-loss rates of first star. Calculations by Muijres et al. (2012) show that, indeed, CNO-burning enriches the mass-loss rate in some cases but this enrichment is not significant for massive stars. For instance, Muijres et al. (2012) have found that very metal-poor stars (with an initial metallicity $Z \leq 10^{-4}$) have no wind, as the high-ionization species of the CNO elements have only few strong lines to drive an outflow. According with the calculations of these authors, the stars with a little higher initial metallicity ($Z < 0.02$–0.04) have the mass-loss rate $\approx$25–50 per cent higher but this $\dot{M}$ still cannot prevent the thermal generation. Therefore, it seems that Population III stars with very low initial metallicity ($Z < 0.02$–0.04) might exhibit the magnetic field generated by the thermal mechanism.

The linear analysis done in this paper allows to account for only B_y component of the magnetic field. The real topology of the magnetic field in early stars is uncertain but it can be rather complex and contain the radial component as it is usually detected in stars. The radial field can be generated because of a number of non-linear effects that accompany a development of the thermomagnetic instability. Also, the radial motions associated with the stellar wind can be important for a generation of the radial magnetic field. A length-scale of the radial field (if it is generated) is far beyond the scope of this paper and present-day observations. It should be noted, however, that recent observations of present-day massive stars show no presence of the large-scale field in their magnetospheres

The thermal generation is determined by the Nernst effect and is caused by the term proportional to $\beta_{\wedge}$ in induction equation (5). Therefore, the generation becomes slower if this term decreases. This type of saturation is efficient if advection of the magnetic field lines by the stellar wind is weak. This condition is satisfied often in laboratory experiments with laser plasma where the Hall parameter caused by the generated field reaches $\sim$1 (Tidman & Shanny 1974; Andrushchenko & Pavlenko 2004; Bissell et al. 2012; Bissell 2015). Perhaps, a similar saturation mechanism operates in neutron stars (Urpin et al. 1986, Wiebicke & Geppert 1992). Therefore, the magnetic field is determined by the estimate $x_{\rm e} = \omega_B \tau_{\rm e} \sim 1$. Likely, this condition yields an estimate of the saturation regime of the magnetic instability in stars with weak stellar wind as well. Then, the condition $x_{\rm e}\omega_B \tau_{\rm e} \sim 1$ yields the following estimate for a saturation magnetic field

$$B_{\rm sat} = \frac{m_{\rm e} c}{e \tau_{\rm e}} \sim 10^2\, \frac{n_{13} \Lambda}{T_4^{3/2}} \quad {\rm G}. \tag{30}$$

Likely, the magnetic field generated in massive stars in the early Universe is of the same order of magnitude as the field of massive stars in the present-day Universe.

By making use of equation (30), one can estimate the characteristic mass-loss rate that prevents the field generation by thermal mechanism in the first stars. A large number of models of Population III stars has been considered by Yoon et al. (2012). For stars with the initial mass $M = 100$–$300\,{\rm M}_{\odot}$, the typical radius is 12–20 ${\rm R}_{\odot}$ and the initial surface temperature is $\sim$40 000–60 000 K. The lifetime of such stars is $(3 - 2) \times 10^6$ yr. Therefore, the thermal generation can operate in such stars if $\dot{M} < 10^{-10}\,{\rm M}_{\odot}\,{\rm yr}^{-1}$. Since the wind is basically weaker in first massive stars, one can expect that the fraction of stars with the magnetic field is higher in Population III than in the present-day massive stars .

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