Astron. Nachr. / AN **337**, No. 6, 633 – 639 (2016) / **DOI** 10.1002/asna.201512349

Chemical spots and oscillatory diffusion modes in magnetic stars

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Received 2015 Jul 30, accepted 2016 Jan 20
Published online 2016 Jul 01

Key words diffusion – magnetohydrodynamics (MHD) – stars: abundances – stars: magnetic fields – starspots

The stars of the middle main sequence often have spot-like chemical structures at their surfaces. We consider diffusion caused by electric currents and argue that such current-driven diffusion can form chemical inhomogeneities in a plasma. The considered mechanism can contribute to a formation of element spots in Hg-Mn and Ap-stars. Due to the Hall effect, diffusion in the presence of electric currents can be accompanied by the propagation of a particular type of magnetohydrodynamic modes in which only the impurity number density oscillates. Such modes exist if the magnetic pressure is much greater than the gas pressure and can be the reason for variations of the abundance peculiarities in stars.

1 Introduction

The stars of the middle main sequence often have relatively quiescent surface layers, but abundance peculiarities can develop in their atmospheres since, in general, there are physical processes that lead to an evolution of the atmospheric chemistry during the main sequence lifetime. These processes, however, are sensitive to the stellar conditions. For example, in cool stars (with the effective surface temperature, $T_{\rm eff} \leq 7000$ K), the gravitational separation is overwhelmed by deep convective mixing of the outer envelope and, likely, does not lead to abundance peculiarities. In hot stars with $T_{\rm eff} \geq 20\,000$ K, the radiation flux drives a strong stellar wind, stripping plasma off the stellar surface so quickly that the processes of separation are not able to compete. It appears that the A- and late B-type main-sequence stars lie between these two values of $T_{\rm eff}$. Chemical composition can evolve in the atmospheres of such stars, for example, because of loss of heavy ions caused by gravitational settling. Also, the atmosphere can acquire ions driven upwards by radiative acceleration due to the radiative energy flux (Michaud et. al. 1976).

Many stars with peculiar chemical abundances show line-profile variations. The generally accepted point of view is that these variation are caused by the rotation of chemical spots at the stellar surface (see, e.g., Pyper 1969; Khokhlova 1985; Silvester et al. 2012). The exact reasons of inhomogeneous surface distributions on stars are unknown. It is often believed that chemical spots can occur in the presence of a strong magnetic field. For example, Ap stars show variations of both spectral lines and magnetic field strength that can be caused by rotation of the chemical and magnetic spots. The reconstruction of the stellar magnetic geometry from observations has been a complex problem for decade. The magnetic Doppler imaging code developed by Piskunov & Kochukhov (2002) makes it possible to derive the magnetic map of a star self-consistently with the distribution of chemical elements. The reconstructions show that the magnetic and chemical maps can be extremely complex (Kochukhov et al. 2004a).

Surprisingly, it turns out that usually chemical elements do not exhibit a clear correlation with the magnetic geometry. For instance, Kochukhov et al. (2004b) have found that almost all elements (except maybe lithium and oxygen) of the Ap-star HR 3831 do not follow the symmetry of the dipolar magnetic field but are distributed in a rather complex manner. The calculated distributions demonstrate the complexity of diffusion in Ap-stars and discard a point of view that diffusion leads to a formation of the chemical spots symmetric with respect to the longitudinal magnetic field (Kochukhov 2004). Likely, chemical distributions are affected by a number of poorly understood phenomena in the surface layers of stars and are not directly related to the strength of the longitudinal field.

Often, the formation of the chemical spots is related to anisotropic diffusion in a strong magnetic field (see, e.g., Michaud 1970). Indeed, the magnetic field of Ap-stars ($\sim 10^3$–10^4 G) can magnetize electrons in a plasma that, generally, leads to anisotropic transport and can produce an inhomogeneous distribution of chemical elements. Anisotropy of diffusion in a magnetized plasma is characterized by the Hall parameter, $x_e = \omega_{Be} \tau_e$, where $\omega_{Be} = eB/m_e c$ is the gyrofrequency of electrons and τ_e is their relaxation time; B is the magnetic field. In a hydrogen plasma, $\tau_{\rm e} = 3 \sqrt{m_e} (k_{\rm B} T)^{3/2} / 4 \sqrt{2\pi} e^4 n \Lambda$ (see, e.g., Spitzer 1978), where n and T are the number density of electrons and their temperature, Λ is the Coulomb logarithm. At $x_e \geq 1$, the rates of diffusion along and across the magnetic

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field become different and, in general, diffusion can result in some inhomogeneous distribution of elements at the surface. The condition $x_e \geq 1$ yields the following estimate of the magnetic field:

$$B \geq B_e = 2.1\times10^3 \Lambda_{10}\, n_{15}\, T_4^{-3/2} \quad \text{G}, \tag{1}$$

where $\Lambda_{10} = \Lambda/10$, $n_{15} = n/10^{15}$, and $T_4 = T/10^4$ K.

Some Ap-stars that exhibit spot-like chemical structures have a sufficiently strong magnetic field that satisfies this condition. Note, however, that the magnetic field (1) magnetizes only electrons and, as a result, its effect on diffusion of heavy ions is relatively weak. For example, the difference of diffusion coefficients along and across the field basically does not exceed $\sim$ 10–15 % in this case. A much stronger field is required to magnetizes protons and to produce strong chemical inhomogeneities in stars. In this case, one requires the Hall parameter for protons, $y = eB\tau_p/m_p c$, to obey $y > 1$. Here $\tau_p = 3\sqrt{m_p}(k_B T)^{3/2}/4\sqrt{2\pi}e^4 n\Lambda$ is the relaxation time for protons (see, e.g., Spitzer 1978). The condition $y > 1$ yields

$$B > B_p = 10^5\, n_{15} T_4^{-3/2} \Lambda_{10} \quad \text{G}. \tag{2}$$

Such a field leads to a higher anisotropy of diffusion, but it is substantially stronger than the field detected at the surface of Ap-stars. Note that the two important factors leading to anisotropic diffusion – magnetization of electrons and protons – are usually neglected in calculations of diffusion in stellar atmospheres (see, e.g., Michaud 1970; Vauclair et al. 1979; Alecian & Stift 2006) and only magnetization of heavy ions is taken into account. The latter requires, however, even a stronger field than that given by Eqs. (1) and (2).

In recent years, the discovery of chemical inhomogeneities in Hg-Mn stars has risen additional doubts regarding the magnetic origin of these inhomogeneities. The inhomogeneous distribution of some chemical elements over the surface of Hg-Mn stars was discussed first by Hubrig & Mathys (1995). In contrast to Ap-stars, no strong large-scale magnetic field has ever been detected in Hg-Mn stars (see, e.g., Wade et al. 2004 who find no longitudinal field above 50 G in the Hg-Mn star α And with a spotted chemical structure). This field is obviously not sufficient to magnetize the plasma.

Weak magnetic fields in the atmospheres of Hg-Mn stars have been detected also by a number of authors (see, e.g., Hubrig & Castelli 2001; Hubrig et al. 2006; Makaganiuk et al. 2011, 2012; Hubrig et al. 2012). On the other hand, magnetic fields up to a few hundred Gauss have been detected in several Hg-Mn stars as well (see, e.g., Mathys & Hubrig 1995). Measurements by Hubrig at al. (2010) reveal a longitudinal magnetic field of the order of a few hundred Gauss in the spotted star AR Aur. The complex interrelations between the magnetic field and element spots clearly indicate how incomplete our understanding of diffusion processes in stars is. The detected field strength often is not sufficient to influence diffusion processes but, nevertheless, chemical inhomogeneities occur in many objects. Most likely, this point indicates that there should exist some additional diffusion mechanisms in stars that can create chemical spots even if the magnetic field is relatively weak.

In this paper, we consider the diffusion process that can contribute to a formation of chemical inhomogeneities in stars and is typical only for plasmas. This process is relevant to electric currents and is well studied in a laboratory plasma (see, e.g., Vekshtein et al. 1975; Vekstein 1987). For example, it plays an important role in some thermonuclear fusion experiments with magnetic confinement because the number density of heavy ions determines the rate of radiative cooling and is crucial for such experiments (see, e.g., Tange 1975). In these devices, the current-driven diffusion causes heavy ions to diffuse away from the hot plasma region (e.g., Markvoort & Rem 1975). Unfortunately, this type of diffusion has not been considered in detail in stellar conditions except in a recent study by Urpin (2015) where the main qualitative features of this process are discussed and a comparison of the diffusion rate caused by electric currents with that of other types of diffusion is made. In combination with other mechanisms, this process can contribute to the formation of chemical spots.

In the present paper, we continue the study of current-driven diffusion in stars. By making use of a simple model, we argue that such diffusion in combination with the Hall effect can be the reason of particular type of modes in stellar atmospheres. In these modes, the impurity number density oscillates alone whereas other parameters of the plasma remain unchanged. These modes can be responsible for variations in the atmospheric abundances of magnetic stars.

2 Diffusion velocity

Consider a cylindrical plasma configuration with the magnetic field parallel to the axis z, $\boldsymbol{B} = B(s)\boldsymbol{e}_z$; (s, φ, z) and $(\boldsymbol{e}_s, \boldsymbol{e}_\varphi, \boldsymbol{e}_z)$ are cylindrical coordinates and the corresponding unit vectors. The electric current in such configuration is

$$j_\varphi = -(c/4\pi)(\mathrm{d}B/\mathrm{d}s). \tag{3}$$

We suppose that $j_\varphi \to 0$ at large s and, hence, $B \to B_0 =$ const at $s \to \infty$. The magnetic geometry of ApBp stars is not well studied and, therefore, we use this simplified model to understand the main qualitative features of the current-driven diffusion mechanism. Note that, in some cases, the considered configuration can mimic real magnetic fields with a high accuracy. This is valid, for example, for the magnetic field near the magnetic pole where field lines are very close to a cylindrical geometry (see, e.g., Urpin & Van Riper 1993).

Note that $B(s)$ can not be an arbitrary function of s because, generally, the magnetic configurations can be unstable for some dependences $B(s)$ (see, e.g., Tayler 1973; Bonanno & Urpin 2008a,b). The characteristic timescale of this instability is usually of the order of the time taken for an Alvén wave to travel around the star, i.e. much shorter than the diffusion timescale. Therefore, any formation of chemical structures in such unstable magnetic configurations seems to be impossible.

We assume that plasma is fully ionized and consists of electrons e, protons p, and a small admixture of heavy ions i. The number density of species i is small and does not influence the dynamics of plasma. Therefore, these ions can be treated as trace particles interacting only with a background hydrogen plasma.

The partial momentum equations in a fully ionized multicomponent plasma have been considered by a number of authors (see, e.g., Urpin 1981). This study deals mainly with a hydrogen-helium plasma. However, the derived equations can be applied for a hydrogen plasma with a small admixture of any other ions if their number density is small. If the mean hydrodynamic velocity of the plasma is zero and only small diffusive velocities are non-vanishing, the partial momentum equation for the species i reads

$$-\nabla p_i + Z_i e n_i \left(\boldsymbol{E} + \frac{\boldsymbol{V}_i}{c} \times \boldsymbol{B} \right) + \boldsymbol{R}_{ie} + \boldsymbol{R}_{ip} + \boldsymbol{F}_i = 0, \quad (4)$$

where Z_i is the charge number of the species i, p_i and n_i are its partial pressure and number density, $\boldsymbol{E}$ is the electric field in plasma, and $\boldsymbol{V}_i$ is the diffusion velocity. Since diffusive velocities are usually very small, we neglect the terms proportional $(\boldsymbol{V}_i \cdot \nabla)\boldsymbol{V}_i$ in the momentum equation (4). For the sake of simplicity, we consider the case T = const and neglect thermodiffusion. The force $\boldsymbol{F}_i$ is the external force on species i; in stellar conditions, $\boldsymbol{F}_i$ is the sum of gravitational and radiative forces. The forces $\boldsymbol{R}_{ie}$ and $\boldsymbol{R}_{ip}$ are caused by the interaction of ions i with electrons and protons, respectively. Note that forces $\boldsymbol{R}_{ie}$ and $\boldsymbol{R}_{ip}$ are internal, but the sum of internal forces over all plasma components is zero in accordance with Newton's third law. If n_i is small compared to the number density of protons, $\boldsymbol{R}_{ie}$ is given by

$$\boldsymbol{R}_{ie} = -(Z_i^2 n_i / n) \boldsymbol{R}_e, \quad (5)$$

where $\boldsymbol{R}_e$ is the force acting on the electron gas (see, e.g., Urpin 1981). Since $n_i \ll n$, $\boldsymbol{R}_e$ is determined mainly by scattering of electrons on protons but scattering on ions i gives a small contribution to $\boldsymbol{R}_e$ as well. Therefore, we can use for $\boldsymbol{R}_e$ the expression for a hydrogen plasma calculated by Braginskii (1965).

Generally, the force $\boldsymbol{R}_e$ contains terms proportional to the temperature gradient and the relative velocity of electrons and protons. In our model, the plasma is assumed to be isothermal, therefore the expression for $\boldsymbol{R}_e$ reads

$$\boldsymbol{R}_e = -\alpha_{\parallel} \boldsymbol{u}_{\parallel} - \alpha_{\perp} \boldsymbol{u}_{\perp} + \alpha_{\wedge} \boldsymbol{b} \times \boldsymbol{u}, \quad (6)$$

where $\boldsymbol{u} = -\boldsymbol{j}/en$ is the current velocity of electrons; $\boldsymbol{b} = \boldsymbol{B}/B$; the subscripts $\parallel$, $\perp$, and $\wedge$ denote the parallel, perpendicular, and the so called Hall components of the corresponding vector; $\alpha_{\parallel}$, $\alpha_{\perp}$, and $\alpha_{\wedge}$ are the coefficients calculated by Braginskii (1965). The force (6) describes a standard friction between different components in a magnetized plasma caused by a relative motion of the electron and proton gases. Taking into account Eq. (3), we have $\boldsymbol{u} = (c/4\pi e n)(\mathrm{d}B/\mathrm{d}s)\, \boldsymbol{e}_{\varphi}$. Since $\boldsymbol{B} \perp \boldsymbol{u}$ in our model, we have $\boldsymbol{u}_{\parallel} = 0$.

In this paper, we consider diffusion only in a relatively weak magnetic field that does not magnetize electrons, $x_e \ll 1$. Substituting Eq. (6) into Eq. (5) and using the coefficients $\alpha_{\perp}$ and $\alpha_{\wedge}$ calculated by Braginskii (1965), we obtain, accurate in linear terms in x_e,

$$R_{ie\varphi} = Z_i^2 n_i \left(0.51 \frac{m_e}{\tau_e} u \right), \quad R_{ies} = Z_i^2 n_i \left(0.21 x \frac{m_e}{\tau_e} u \right). \quad (7)$$

If T = const, the friction force $\boldsymbol{R}_{ip}$ is proportional to the relative velocity of ions i and protons. Like $\boldsymbol{R}_e$ (see Eq. 6), this force also has a tensor character and, generally, depends on the magnetic field. In the presence of the magnetic field, it can be represented in the form

$$\boldsymbol{R}_{ip} = \alpha_{\parallel}^{(W)} (\boldsymbol{V}_p - \boldsymbol{V}_i)_{\parallel} + \alpha_{\perp}^{(W)} (\boldsymbol{V}_p - \boldsymbol{V}_i)_{\perp} + \alpha_{\wedge}^{(W)} \boldsymbol{b} \times (\boldsymbol{V}_p - \boldsymbol{V}_i), \quad (8)$$

where the coefficients $\alpha_{\parallel}^{(W)}$, $\alpha_{\perp}^{(W)}$, and $\alpha_{\wedge}^{(W)}$ have been calculated by Urpin (1981). However, the dependence of these coefficient on the magnetic field becomes important only if the magnetic field is of the order of or greater than $B_i = B_p Z_i^2$. The field B_i is always greater than B_e (Eq. 1) and B_p (Eq. 2) which is qualitatively clear because it is more difficult to magnetize heavy ions with $Z_i > 1$ than protons or electrons. Surprisingly, only this effect of the magnetic field on the anisotropy of diffusion processes is usually taken into account in calculations of chemical inhomogeneities in stars (see, e.g., Michaud et al. 1976; Vauclair et al. 1979; Alecian & Stiff 2006). The effects associated with $\boldsymbol{R}_e$ are usually neglected but we will show that they are important as well, particularly in stars without very strong magnetic fields.

The force $\boldsymbol{R}_{ip}$ has an especially simple shape if $A_i = m_i/m_p \gg 1$ (see Urpin 1981), and we consider only this case. We neglect the influence of the magnetic field on $\boldsymbol{R}_{ip}$ since this influence becomes important only in a strong magnetic field $\geq B_p$. Taking into account that the velocity of the background plasma is zero, $\boldsymbol{V}_p = 0$, the friction force $\boldsymbol{R}_{ip}$ can be written as

$$\boldsymbol{R}_{ip} = (0.42 m_i n_i Z_i^2 / \tau_i), (-\boldsymbol{V}_i), \quad (9)$$

where $\tau_i = 3\sqrt{m_i}(k_B T)^{3/2} / 4\sqrt{2\pi} e^4 n \Lambda$, and τ_i / Z_i^2 is the timescale of ion-proton scattering. We assume that Λ is the same for all types of scattering (see, e.g., Urpin 1981).

The cylindrical components of Eq. (4) yield

$$-\frac{\mathrm{d}}{\mathrm{d}s}(n_i k_{\mathrm{B}} T) + Z_i e n_i \left(E_s + \frac{V_{i\varphi}}{c} B \right) + R_{ies} + R_{ips} = 0, \quad (10)$$

$$Z_i e n_i \left(E_{\varphi} - \frac{V_{is}}{c} B \right) + R_{ie\varphi} + R_{ip\varphi} = 0, \quad (11)$$

$$-\frac{\mathrm{d}}{\mathrm{d}z}(n_i k_{\mathrm{B}} T) + Z_i e n_i E_z + R_{iez} + R_{ipz} + F_{iz} = 0. \quad (12)$$

In our simplified magnetic configuration, we have $R_{iez} = 0$. Eqs. (10)–(12) depend on cylindrical components of the electric field, E_s, E_{φ}, and E_z. These components can be determined from the momentum equations for electrons and protons,

$$-\nabla(nk_{\mathrm{B}}T) - en\left(\boldsymbol{E} + \frac{\boldsymbol{u}}{c}\times\boldsymbol{B}\right) + \boldsymbol{R}_e + \boldsymbol{F}_e = 0, \quad (13)$$

$$-\nabla(nk_{\mathrm{B}}T) + en\boldsymbol{E} - \boldsymbol{R}_e + \boldsymbol{F}_p = 0. \quad (14)$$

In these equations, we neglect collisions of electrons and protons with the ions i since these ions are considered as the test particles and their number density is assumed to be small.

The sum of Eqs. (13) and (14) yield the equation of hydrostatic equilibrium. The difference of Eqs. (14) and (13) yields the following expression fo the electric field:

$$\boldsymbol{E} = -\frac{1}{2}\frac{\boldsymbol{u}}{c}\times\boldsymbol{B} + \frac{\boldsymbol{R}_e}{en} - \frac{1}{2en}(\boldsymbol{F}_p - \boldsymbol{F}_e). \quad (15)$$

Taking into account the friction force $\boldsymbol{R}_e$ (Eq. 6) and the coefficients $\alpha_\perp$ and $\alpha_\wedge$ calculated by Braginskii (1965), we obtain with accuracy in linear terms in x_e

$$E_s = -\frac{uB}{2c} - \frac{1}{e}\left(0.21\frac{m_e u}{\tau_e}x_e\right), \quad E_\varphi = -\frac{1}{e}\left(0.51\frac{m_e u}{\tau_e}\right),$$
$$E_z = -\frac{1}{2en}(F_{pz} - F_{ez}). \quad (16)$$

Substituting Eqs. (5) and (16) into the vertical component of the momentum equation (12), we obtain the following expression for the velocity of vertical diffusion:

$$V_{iz} = -D\frac{\mathrm{d}\ln n_i}{\mathrm{d}z} + \frac{D}{n_i k_{\mathrm{B}}T}F_z^{(i)}, \quad (17)$$

where $D = 2.4c_i^2\tau_i/Z_i^2$ is the diffusion coefficient, $c_i^2 = k_{\mathrm{B}}T/m_i$, and

$$F_z^{(i)} = F_{iz} - \frac{Z_i n_i}{2n}(F_{pz} - F_{ez}). \quad (18)$$

Usually, radiative acceleration due to the radiative energy flux and gravitational settling gives the main contribution to the external force $F_z^{(i)}$ (Michaud et al. 1976). The diffusion velocity caused by these forces can be relatively fast and, therefore, the vertical diffusion often is faster than diffusion in the tangential direction parallel to the surface. As a result, the vertical distribution of chemical elements reaches a quasi-steady equilibrium on a relatively short timescale. Note, however, that expression (17) for V_{iz} does not depend on the magnetic field in our approximation ($x_e \ll 1$) and, therefore, the vertical diffusion cannot form chemical spots if the magnetic field is relatively weak and $B \ll B_e$. This conclusion is obvious in our approach because we neglect the magnetization of heavy ions deriving Eq. (17). The vertical diffusion can form chemical spots only if V_{iz} depends on tangential coordinates. Such departures from spherical symmetry can appear in stars, for example, if one take into account small corrections to $\alpha_\parallel^{(W)}$ and $\alpha_\perp^{(W)}$ caused by the magnetic field. However, these corrections are of the order of $(\omega_{Bi}\tau_i)^2$ where ω_{Bi} is the cyclotron frequency of ions and they are small if $B \ll B_p/Z_i^2$ (see Eq. 2). This condition is satisfied in all Ap-stars and, therefore, the rate of formation of element spots by vertical diffusion is much smaller than the rate of vertical diffusion itself. Since the vertical diffusion is considered in detail by a number of authors, we concentrate mainly on the horizontal diffusion.

The tangential components of the diffusion velocity can be obtained from Eqs. (10) and (11). Taking into account Eq. (9) for $\boldsymbol{R}_{ip}$, one can transform Eqs. (10) and (11) into

$$V_{is} - qV_{i\varphi} = A, \quad V_{i\varphi} + qV_{is} = G, \quad (19)$$

where

$$A = \frac{D}{n_i k_{\mathrm{B}}T}\left(-\frac{\mathrm{d}p_i}{\mathrm{d}s} + Z_i e n_i E_s + R_{ies}\right), \quad (20)$$

$$G = \frac{D}{n_i k_{\mathrm{B}}T}\left(Z_i e n_i E_\varphi + R_{ie\varphi}\right), \quad q = 2.4\frac{eB}{Z_i m_i c}\tau_i. \quad (21)$$

Then the diffusion velocities in the s- and φ-directions are

$$V_{is} = \frac{A + qG}{1 + q^2}, \quad V_{i\varphi} = \frac{G - qA}{1 + q^2}. \quad (22)$$

The parameter q is of the order of $\omega_{Bi}\tau_i$ and is small even for magnetic fields typical for Ap-stars. Then, we have for $q \ll 1$

$$V_{is} \approx A, \quad V_{i\varphi} \approx G. \quad (23)$$

Substituting Eqs. (7) and (16) into expressions (20)–(21) for A and G, we obtain the following expressions for the diffusion velocities:

$$V_{is} = V_{n_i} + V_B, \quad V_{n_i} = -D\frac{\mathrm{d}\ln n_i}{\mathrm{d}s}, \quad V_B = D_B\frac{\mathrm{d}\ln B}{\mathrm{d}s}, \quad (24)$$

$$V_{i\varphi} = D_{B\varphi}\frac{\mathrm{d}B}{\mathrm{d}s}; \quad (25)$$

V_{ni} is the velocities of ordinary diffusion and V_B is the diffusion velocity caused by the electric current. The corresponding diffusion coefficients are

$$D = \frac{2.4c_i^2\tau_i}{Z_i^2}, \quad D_B = \frac{2.4c_{\mathrm{A}}^2\tau_i}{Z_i A_i}(0.21Z_i - 0.71), \quad (26)$$

$$D_{B\varphi} = 1.22\sqrt{\frac{m_e}{m_i}}\frac{c(Z_i - 1)}{4\pi e n Z_i}. \quad (27)$$

where $c_i^2 = k_{\mathrm{B}}T/m_i$ and $c_{\mathrm{A}}^2 = B^2/(4\pi n m_p)$. Equations (24)–(25) describe the drift of ions i under the combined influence of ∇n_i and $\boldsymbol{j}$. Note that heavy ions diffuse not only in the radial direction but also rotate around the magnetic axis.

3 Distribution of ions caused by current-driven diffusion

The condition of hydrostatic equilibrium in our model reads

$$-\nabla p + \boldsymbol{F} + \frac{1}{c}\boldsymbol{j}\times\boldsymbol{B} = 0, \quad (28)$$

where p is the gas pressure, ρ is the density, and $\boldsymbol{F}$ is an external force acting on the plasma. Since the background plasma is fully ionized hydrogen, $p \approx 2nk_{\mathrm{B}}T$, where k_{B} is the Boltzmann constant. Under stellar conditions, $\boldsymbol{F}$ is usually the sum of two forces, $\boldsymbol{F} = \boldsymbol{F}_{\mathrm{g}} + \boldsymbol{F}_{\mathrm{rad}}$, where $\boldsymbol{F}_{\mathrm{g}} = \rho\boldsymbol{g}$ is the gravity force, and $\boldsymbol{F}_{\mathrm{rad}}$ is caused by radiative acceleration due to the radiative energy flux from the interior. We assume that the two external forces $\boldsymbol{F}_{\mathrm{g}}$ and $\boldsymbol{F}_{\mathrm{rad}}$ act in the vertical direction. Then, the z-component of Eq. (28) determines the vertical distribution of a background plasma and reads $\partial p/\partial z = F_z$. The s-component of these equation describes the transverse structure of a magnetic atmosphere. Integrating the s-component of Eq. (28), we obtain

$$n = n_0 \left(1 + \beta_0^{-1} - \beta^{-1}\right), \tag{29}$$

where $\beta = 8\pi p_0/B^2$; (p_0, n_0, T_0, β_0) are values of the corresponding quantities at $s \to \infty$.

Consider the equilibrium distribution of heavy ions in our model. In equilibrium, we have $V_{is} = 0$, and Eq. (24) yields

$$D \frac{\mathrm{d} \ln n_i}{\mathrm{d}s} = D_B \frac{\mathrm{d} \ln B}{\mathrm{d}s}. \tag{30}$$

The term on the r.h.s. describes the effect of electric currents on the distribution of impurities. Note that this type of diffusion is driven by the electric current rather than by an inhomogeneity of the magnetic field. The conditions $\mathrm{d}B/\mathrm{d}s \neq 0$ and $j \neq 0$ are equivalent in our simplified magnetic configuration. Equation (28) yields

$$\frac{\mathrm{d}}{\mathrm{d}s}(n k_{\mathrm{B}} T) = -\frac{B}{8\pi} \frac{\mathrm{d}B}{\mathrm{d}s}. \tag{31}$$

Substituting Eq. (31) into Eq. (30) and integrating, we obtain

$$\frac{n_i}{n_{i0}} = \left(\frac{n}{n_0}\right)^{\mu}, \tag{32}$$

where

$$\mu = -2Z_i\,(0.21Z_i - 0.71) \tag{33}$$

and n_{i0} the value of n_i at $s \to \infty$. Denoting the local abundance of the element i as $\gamma_i = n_i/n$ and taking into account Eq. (29), we have

$$\frac{\gamma_i}{\gamma_{i0}} = \left(\frac{n}{n_0}\right)^{\mu-1} = \left(1 + \frac{1}{\beta_0} - \frac{1}{\beta}\right)^{\mu-1}, \tag{34}$$

where $\gamma_{i0} = n_{i0}/n_0$. Local abundances turn out to be flexible to the field strength, and this concerns ions with large charge numbers in particular. The exponent $(\mu-1)$ can reach large negative values for elements with large Z_i and, hence, produce strong abundance anomalies. For instance, $(\mu-1)$ is equal 0.16, –0.52, and –2.04 if Z_i =2, 3, and 4, respectively. Note that $(\mu-1)$ changes its sign as Z_i increases: $(\mu-1) > 0$ if $Z_i = 2$ but $(\mu-1) < 0$ for $Z_i \geq 3$. Therefore, elements with $Z_i \geq 3$ are in deficit ($\gamma_i < \gamma_{i0}$) in the region with a weak magnetic field ($B < B_0$), but these elements should be overabundant in the spot where the magnetic field is stronger than the external field B_0.

Note that the dependence of the exponent $(\mu - 1)$ on Z_i can be responsible for the increase of the helium abundance in magnetic stars with stellar age. This increase was first discovered by Bailey et al. (2014) and is very unexpected within the framework of the standard theory because radiative levitation of He is very weak and becomes weaker as the star evolves. However, the increase of the helium abundance seems to be rather natural if one takes into account the current-driven diffusion. Indeed, observations indicate that the magnetic field decreases with the stellar age (see, e.g., Bailey et al. 2014) because of Ohmic dissipation and, hence, a contrast between the magnetic spots and the ambient plasma becomes weaker. As it follows from Eq. (34), a weaker contrast of the magnetic field leads to a higher local abundance of He in a spot.

It is generally believed that standard diffusion is smoothing chemical inhomogeneities on a timescale of the order of L^2/D, where L is the length scale of a non-uniformity. However, this is not the case for a chemical distribution given by Eq. (34) which can exist during a much longer time than $\sim L^2/D$. In our model, the distribution (34) is reached due to the balance of two diffusion processes, standard ($\propto \nabla n_i$) and current-driven ($\propto \mathrm{d}B/\mathrm{d}s$) diffusion which pushes heavy ions in the opposite directions. As a result, $V_{is} = 0$ in the equilibrium state and this state can be maintained as long as the electric currents exist. Therefore, the characteristic lifetime of chemical structures is of the order of the decay time of electric currents, i.e. determined by Ohmic dissipation, and is $\sim 4\pi\sigma L^2/c^2$, where σ is the electrical conductivity. The decay of the magnetic field is very slow under stellar conditions, and the decay timescale can be longer than the diffusion timescale if $D > c^2/4\pi\sigma$. Under such conditions, the lifetime of a spot is entirely determined by the Ohmic decay time.

Note that $V_{is} = 0$ in the equilibrium state but the φ-component of the diffusion velocity is non-zero. It turns out that impurities rotate around the magnetic axis even if equilibrium is reached, $V_{i\varphi} \neq 0$. The direction of rotation depends on the sign of $\mathrm{d}B/\mathrm{d}s$ and is opposite to the electric current. Since electrons move in the same direction, heavy ions turn out to be carried along electrons. Different ions move with different velocities around the axis, and the difference between different sorts of ions, $\Delta V_{i\varphi}$, is of the order of

$$\Delta V_{i\varphi} \sim \frac{c}{4\pi e n} \sqrt{\frac{m_e}{m_i}} \frac{\mathrm{d}B}{\mathrm{d}s} \sim 3\times 10^{-3} \frac{B_4}{n_{14}\, L_{10}\, A_i^{1/2}} \quad \mathrm{cm\,s^{-1}}, \tag{35}$$

where $B_4 = B/10^4$ G, $n_{14} = n/10^{14}$ cm^{-3}, and $L_{10} = L/10^{10}$ cm. Since different impurities rotate around the magnetic axis with different velocities, the periods of such rotations are also different for different ions. The difference in periods can be estimated as

$$\Delta P = \frac{2\pi L}{\Delta V} \sim 10^6 \frac{L_{10}^2\, n_{14}\, A_i^{1/2}}{B_4} \quad \mathrm{yr}. \tag{36}$$

If the distribution of impurities is non-axisymmetric then such diffusion in the azimuthal direction should lead to slow variations in the abundance peculiarities.

4 Oscillatory diffusion modes

In our model of plasma with a cylindrical symmetry, the continuity equation for ions i reads

$$\frac{\partial n_i}{\partial t} - \frac{1}{s}\frac{\partial}{\partial s}\left(sD\frac{\partial n_i}{\partial s} - s n_i \frac{D_B}{B}\frac{\mathrm{d}B}{\mathrm{d}s}\right) = 0. \tag{37}$$

Together with Eqs. (24)–(25), this equation describes the diffusion of ions i in the presence of electric currents.

Let us assume that the plasma is in a diffusion equilibrium (Eq. 30) and, hence, the distribution of elements in such a basic state is given by Eqs. (34). Consider the behaviour of small disturbances of the number density of impurity from this equilibrium by making use of a linear analysis of Eq. (37). Since the number density of impurity i is small, its influence on parameters of the basic state is negligible. For the sake of simplicity, we assume that small disturbances are axisymmetric and do no depend on the vertical coordinate, z. Such disturbances have a shape of cylindrical waves. Denoting disturbances of the impurity number density by δn_i and linearizing Eq. (37), we obtain the equation governing the evolution of such small disturbances,

$$\frac{\partial \delta n_i}{\partial t} - \frac{1}{s}\frac{\partial}{\partial s}\left(sD\frac{\partial \delta n_i}{\partial s} - s\delta n_i \frac{D_B}{B}\frac{\mathrm{d}B}{\mathrm{d}s}\right) = 0. \quad (38)$$

For the purpose of illustration, we consider only disturbances with a wavelength shorter than the length scale of unperturbed quantities. In this case, we can use the so called local approximation and assume that small disturbances are $\propto \exp(-\mathrm{i}ks)$, where k is the wavevector, $ks \gg 1$. Since the basic state does not depend on time, δn_i can be represented as $\delta n_i \propto e^{\mathrm{i}\omega t - \mathrm{i}ks}$, where ω should be calculated from the dispersion equation. Substituting δn_i in such form into Eq. (38), we obtain the following dispersion equation:

$$\mathrm{i}\omega = -\omega_R + \mathrm{i}\omega_I, \quad \omega_R = Dk^2, \quad \omega_\mathrm{I} = kD_B\,(\mathrm{d}\ln B/\mathrm{d}s). \quad (39)$$

This dispersion equation describes a particular type of cylindrical waves in which only the number density of impurities oscillates. The quantity ω_R characterizes decay of these waves with the characteristic timescale $\sim (Dk^2)^{-1}$ which is typical for standard diffusion. The frequency ω_I describes oscillations of impurities caused by the Lorentz force. Note that the frequency can be of any sign but ω_R is always positive. The diffusion waves are aperiodic if $\omega_R > |\omega_\mathrm{I}|$ and oscillatory if $|\omega_\mathrm{I}| > \omega_R$. The latter condition is equivalent to

$$c_\mathrm{A}^2/c_i^2 > (A_i/Z_i)\,|0.21Z_i - 0.71|^{-1}kL, \quad (40)$$

where $L = |\mathrm{d}\ln B/\mathrm{d}s|^{-1}$ is the length scale of the magnetic field. Diffusion waves become oscillatory if the field is sufficiently strong and the magnetic pressure is greater than the gas pressure. The frequency of diffusion waves is higher in the region where the magnetic field has a strong gradient. The order of magnitude estimate of ω_I is given by

$$\omega_\mathrm{I} \sim kc_\mathrm{A}(1/Z_iA_i)(c_\mathrm{A}/c_i)(l_i/L), \quad (41)$$

where $l_i = c_i\tau_i$ is the mean free-path of impurity ions. Note that different impurities oscillate with different frequencies. Therefore, the local abundances of different elements can exhibit variations with the time. The characteristic timescale of these variations is shorter in plasma with a stronger magnetic field.

5 Conclusion

We have considered diffusion of elements under a combined influence of standard and current-driven diffusion mechanisms. A diffusion velocity caused by the electric current can be estimated as

$$V_B \sim c_\mathrm{A}(c_\mathrm{A}/c_i)(1/Z_iA_i)(l_i/L) \quad (42)$$

if the magnetic field is relatively weak and electrons are not magnetized. Generally, this velocity can be comparable to velocities caused by other diffusion mechanisms (Urpin 2015). The current-driven mechanism can form chemical inhomogeneities in a plasma even if the magnetic field is weak ($\sim$10–100 G) whereas other diffusion processes require a substantially stronger magnetic field (see Eqs. 1 and 2). Using Eq. (24), the velocity of current-driven diffusion can be estimated as

$$V_B \sim 1.1\times10^{-2}A_i^{-1/2}B_4^2 n_{14}^{-2}T_4^{3/2}\Lambda_{10}L_{10}^{-1} \quad \mathrm{cm\,s^{-1}}, \quad (43)$$

where $\Lambda_{10} = \Lambda/10$. The velocity V_B turns out to be sensitive to the field ($\propto B^2$) and, therefore, diffusion in a weak magnetic field requires a longer time to reach the equilibrium abundances (34).

The current-driven mechanism leads to a drift of ions in the direction perpendicular to both the magnetic field and electric current. Therefore, the distribution of chemical elements in a plasma depends essentially on the geometry of fields and currents. The mechanism considered can operate both in laboratory plasmas and in various astrophysical bodies where the electric currents are non-vanishing.

Our study reveals that a particular type of magnetohydrodynamic modes exists in a multicomponent plasma in the presence of electric currents. These modes are characterized by oscillations of the impurity number density and exist only if the magnetic pressure exceeds essentially the gas pressure. The frequency of such waves is given by Eq. (39) and turns out to be relatively small. The frequency of diffusion modes is different for different impurities. Therefore, generation of such modes should lead to slow variations of relative abundances of different elements in chemical spots.

The considered mechanism does not depend on the nature of electric currents and can operate if the current is maintained by some mechanism or if it is of fossil origin. If the length scale of the field is L, the Ohmic decay timescale is $t_\mathrm{d} \sim 4\pi\sigma L^2/c^2$, where σ is the conductivity. In subphotospheric layers, we can estimate $\sigma \sim 3\times10^{14}$ s^{-1} and $t_\mathrm{d} \sim 10^7 L_{10}^2$ yr. The timescale of diffusion from subphotospheric layers is $t_B \sim H/V_B$, where H is the scale height. Using Eq. (43) and assuming $B \sim 100$ G, we obtain $t_B \sim 3\times10^6 H_8 L_{10}$ yr, where $H_8 = H/10^8$ cm. Hence, the current-driven diffusion is faster than the Ohmic dissipation if $L_{10} > 1$ and it can form the observed chemical inhomogeneities even if the magnetic field is of the fossil origin.

Note that several studies failed to detect significant global magnetic field in some spotted Hg-Mn stars. For instance, Kochukhov et al. (2011) found an upper limit of only 3 G for the mean longitudinal magnetic field of μ Lep and concluded that formation of chemical spots in Hg-Mn stars is not magnetically driven. Recent observations by Kochukhov et al. (2013) provide an upper limit on the possible magnetic field of HD 65949 of the order of $\sim$ 3–6 G. These limits seems to be much lower than that predicted by Eq. (1). However, these observations reveal only the mean longitudinal component of the magnetic field. Indeed, this component can generally be weak in some stars. However, the considered mechanism does not require a strong longitudinal magnetic field for formation of spots. As it is seen from the above discussion, the current-driven diffusion can operate even if the longitudinal component is vanishing. The observational upper limits on the mean magnetic field modulus are not so stringent as those on the longitudinal component. For example, Kochukhov et al. (2013) found an upper limit of $\sim$ 200–700 G for the magnetic field modulus of HD 65949. This field strength is enough for formation of a chemical spot by the current-driven diffusion. Note that the toroidal field in stars can form configurations with a vanishing longitudinal component. Such configurations can be formed in the surface layers of stars by unstructured or tangled fields as well.

Like other diffusion processes, the current-driven diffusion can lead to the formation of chemical spots and their variability if the star has relatively quiescent surface layers. This condition is fulfilled in various type of stars and, therefore, the current-driven diffusion can manifest itself in different astrophysical bodies. For example, this mechanism can contribute to the formation of element spots in Ap-stars. The magnetic fields have been detected in many of such spotted stars and, likely, these magnetic fields are maintained by electric currents located in the surface layers.

Quiescent surface layers may exist in other types of stars as well, for example, in white dwarfs and neutron stars. Many neutron stars have strong magnetic fields and, most likely, the topology of these fields is very complex with spot-like structures at the surface (see, e.g., Bonanno et al. 2005, 2006). As shown in the present paper, such magnetic configurations can be responsible for the formation of a spot-like element distribution at the surface. Such chemical structures can be important, for instance, for the emission spectra, diffusive nuclear burning (Brown et al. 2002; Chang & Bildsten 2004), etc. The evolution of neutron stars is very complicated, particularly, in binary systems (see, e.g., Urpin et al. 1998a,b) and, as a result, the surface chemistry can be complicated as well. Diffusion processes may play an important role in this chemistry.

Acknowledgements. The author thanks the Russian Academy of Sciences for financial support under the programme OFN-15.

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