Physics of neutron star surface layers and their thermal radiation

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Abstract. I briefly review the physical properties of neutron star surface layers, important for the stellar thermal radiation, taking into consideration the effects of strong magnetic fields.

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INTRODUCTION

Envelopes of neutron stars (in the wide sense) comprise the stellar atmosphere, ocean, outer and inner crusts, and possibly mantle (e.g., [1], and references therein). Although the envelopes constitute less than 10% of the mass M of a typical (with $M \sim 1-2\,M_\odot$ and radius $R \sim 10-15$ km) neutron star, they play an important role. For example, neutrino emission in the crust affects the initial (~ 100 yr) thermal relaxation of the star [2]. Depending on the heat capacity and thermal conductivity in the crust, this initial relaxation can leave an imprint on the subsequent cooling history (e.g., [3]). The shear viscosity of the crust affects localization and damping of stellar oscillations (e.g., [4], and references therein). The glitches of radio pulsars can be associated with depinning of superfluid vortices in the crust (see, e.g., [5], for review).

The *outer* envelopes (envelopes in the restricted sense) lie at densities smaller than the neutron drip density $\rho_{\rm ND} \approx 4\times10^{11}$ g cm^{-3}. In a typical neutron star, the outer envelopes are several hundred meters thick and constitute $\lesssim 10^{-4}$ of the stellar mass (see, e.g., [1]). This thin layer of a dense plasma provides thermal insulation of the stellar interior and controls thus the cooling of a neutron star and its photon luminosity. Furthermore, the properties of the atmosphere (only a few centimeters thick) or the condensed surface (see below) determine the spectrum, beaming, and polarization of emitted radiation.

Apart from its importance for neutron star modeling, the physics of the outer envelopes of neutron stars is interesting on its own. Because the values of temperatures T and densities ρ in the largest part of these envelopes cannot be reached in terrestrial experiments, a comparison of observations of neutron star thermal radiation with theoretical predictions can be invaluable for the physics of dense plasmas. Moreover, most of the currently known neutron stars possess strong magnetic fields B, which cannot be created in laboratory. Therefore, some unique magnetic phenomena, that occur in the neutron star envelopes, should be taken into account in neutron star models and can potentially be revealed by observations.

I briefly review astrophysically important physical properties of matter in the outer envelopes of neutron stars, focusing on surface layers responsible for thermal emission.

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NONMAGNETIZED OUTER ENVELOPES

Fully ionized plasma

Sufficiently deep layers of neutron star envelopes are fully ionized. This regime can be defined by the requirement that the spacing $a_\mathrm{i} = [3/(4\pi n_\mathrm{i})]^{1/3}$ between ions is small compared to the radius of the atomic core, which at $B = 0$ leads to the condition [6] $\rho \gg \rho_\mathrm{eip} = m_\mathrm{i} Z/a_0^3 \sim 10AZ$ g cm^{-3}. Here n_i is the ion number density, a_0 is the Bohr radius, $m_\mathrm{i} = Am_\mathrm{u}$ is the ion mass, m_u is the atomic mass unit, A and Z are the ion mass and charge numbers. At $\rho \gg \rho_\mathrm{eip}$, a model of an *electron-ion plasma* of bare pointlike nuclei on an electron-liquid background can provide a good approximation for thermodynamic functions. Generally, the background is compressible, and Coulomb correlation of ions and electrons should be taken into account.

The state of a free electron gas is determined by the electron number density n_e and temperature T. It is convenient to introduce the parameter $x_\mathrm{r} \equiv p_\mathrm{F}/m_e c \approx 1.009\,(\rho_6 \langle Z\rangle/\langle A\rangle)^{1/3}$, where $p_\mathrm{F} = \hbar(3\pi^2 n_e)^{1/3}$ is the electron Fermi momentum, $\rho_6 \equiv \rho/10^6$ g cm^{-3}, and $\langle\ldots\rangle$ means the average over number fractions of ion species. The electron gas is nonrelativistic at $T \ll T_\mathrm{r}$ and $x_\mathrm{r} \ll 1$ and ultrarelativistic at $x_\mathrm{r} \gg 1$ or $T \gg T_\mathrm{r}$, where $T_\mathrm{r} = m_e c^2/k_\mathrm{B} \approx 5.93 \times 10^9$ K and k_B is the Boltzmann constant,

The strength of the Coulomb interaction of electrons and ions in a plasma can be characterized by the electron and ion Coulomb coupling parameters:

$$\Gamma_e = \frac{e^2}{a_e k_\mathrm{B} T} \approx \frac{22.75}{T_6}\left(\rho_6 \frac{Z}{A}\right)^{1/3}, \qquad \Gamma = \frac{(Ze)^2}{a_\mathrm{i} k_\mathrm{B} T}, \qquad \langle\Gamma\rangle = \Gamma_e \langle Z^{5/3}\rangle, \tag{1}$$

where $T_6 \equiv T/10^6$ K and $a_e = [3/(4\pi n_e)]^{1/3}$ is the mean distance between electrons.

The averaging in the last equality in (1) is justified by the "linear mixing law" (e.g., [7, 8]). However, the case of a mixture of ions with strongly different Z, corresponding to large and small partial Γ values, is more complicated, as recently shown in [9].

The electron degeneracy is usually characterized by the parameter $\theta = T/T_\mathrm{F}$, where $T_\mathrm{F} = T_\mathrm{r}\,(\gamma_\mathrm{r} - 1)$ is the electron Fermi temperature (the Fermi energy in temperature units) and $\gamma_\mathrm{r} = \sqrt{1 + x_\mathrm{r}^2}$. In the non-relativistic and ultrarelativistic cases ($x_\mathrm{r} \ll 1$ and $x_\mathrm{r} \gg 1$), at $B = 0$, this parameter equals $\theta = 0.543\,r_s/\Gamma_e$ and $\theta = (263\,\Gamma_e)^{-1}$, respectively.

Another important parameter is the screening length. In the linear plasma response approximation, the length r_e of *electron* screening is $r_e = \left(4\pi e^2\, \partial n_e/\partial\mu_e\right)^{-1/2}$, where μ_e is the electron chemical potential. For $T \gg T_\mathrm{F}$, r_e equals the Debye length for the electrons: $r_e \approx a_e/\sqrt{3\Gamma_e}$. For $T \ll T_\mathrm{F}$, r_e^{-1} equals the Thomas-Fermi wave number. In this case, $r_e = (\alpha_\mathrm{f}\gamma_\mathrm{r} x_\mathrm{r}/\pi)^{-1/2}\hbar/(2m_e c) \approx 5.4\,\sqrt{x_\mathrm{r}/\gamma_\mathrm{r}}\,a_e$, where $\alpha_\mathrm{f} = e^2/(\hbar c) \approx \frac{1}{137}$. The *ion* screening at $\Gamma \ll 0.1$ is determined by the Debye radius $r_\mathrm{D} = (\langle 3\Gamma/a_\mathrm{i}^2\rangle)^{-1/2}$. However, at $\Gamma \gtrsim 0.1$ the Debye theory does not apply. In this case one can use an *effective* screening length r_i [10, 11], which equals r_D at $\Gamma \ll 1$ and is $\sim a_\mathrm{i}$ at $1 \lesssim \Gamma \lesssim 100$. The *total* effective screening length in the electron-ion fluid equals $r_\mathrm{s} = (r_e^{-2} + r_\mathrm{i}^{-2})^{-1/2}$.

At sufficiently strong coupling between the ions, their infinite motion is replaced by oscillations near equilibrium positions: the plasma solidifies. For a typical middle-age neutron star, this occurs at $\rho \gtrsim 10^6$ g cm^{-3}. In the rigid electron background model, the liquid-solid phase transition occurs at $\Gamma = 175$, but the electron response (screening) can shift this value by tens per cent [12].

The effects of collective oscillations of plasma particles are characterized by the electron and ion plasma frequencies – $\omega_{pe} = (4\pi e^2 n_e/m_e^*)^{1/2}$ and $\omega_{pi} = (4\pi e^2 n_i/m_i^*)^{1/2}$, respectively, where $m_e^* = m_e\gamma_r$ is the effective dynamical mass of an electron on the Fermi surface. The corresponding energies are $\hbar\omega_{pe} \approx 28.437(x_r^{3/2}/\gamma_r^{1/2})$ keV and $\hbar\omega_{pi} = 0.675\,[(\rho_6/\langle A\rangle)\langle Z^2/A\rangle]^{1/2}$ keV.

At $k_B T \lesssim \hbar\omega_{pi}$, the quantization of ion motion is very important. Moreover, for light elements (H and He) the zero-point quantum vibrations of ions may hamper lattice ordering, and one gets the *quantum liquid* instead of the Coulomb crystal (e.g., [13], and references therein; see also a discussion and references in [1]).

Thermodynamic functions of the fully ionized electron-ion plasma can be presented as sums of the terms corresponding to the contributions of the ideal ion (i) and electron (e) gases, the exchange-correlation (xc) contribution for the electrons, the ion-ion (ii) nonideal part (which includes the Coulomb correlations), and the contribution due to the ion-electron (ie) interactions (at arbitrary θ, x_r, and Γ these terms can be calculated using analytical fitting expressions [1]). For example, the pressure is represented as

$$P = P_{id}^{(i)} + P_{id}^{(e)} + P_{xc} + P_{ii} + P_{ie}. \qquad (2)$$

For a strongly degenerate ($T \ll T_F$) and fully ionized plasma, the main contribution in Eq. (2) comes from the term

$$P_{id}^{(e)} = \frac{P_r}{8\pi^2}\left[x_r\left(\frac{2}{3}x_r^2 - 1\right)\gamma_r + \ln(x_r+\gamma_r) + \frac{4\pi^2}{9}\left(\frac{T}{T_r}\right)^2 x_r(\gamma_r + \gamma_r^{-1})\right] \approx \frac{P_r x_r^{3\gamma_{ad}}}{9\pi^2\gamma_{ad}}, \qquad (3)$$

where $P_r \equiv m_e c^2/\bar{\lambda}_C^3 = 1.4218\times10^{25}$ dyn cm^{-2} is the relativistic unit of the electron pressure, $\bar{\lambda}_C \equiv \hbar/(m_e c) = 386.16$ fm is the electron Compton wavelength, and γ_{ad} is an effective adiabatic index, equal to $5/3$ for $x_r \ll 1$ and $4/3$ for $x_r \gg 1$.

However, the leading contribution to the heat capacity C in the important $\rho - T$ domain $\hbar\omega_{pi} \lesssim k_B T \ll k_B T_F$ is provided by the ions: $C \approx C_{id}^{(i)} + C_{ii}$.

The calculation of *radiative opacities* of a fully ionized plasma is relatively simple: one has to take into account only the Thomson (Compton) scattering and free-free transitions. For the latter transitions, calculations of the cross sections beyond the Born approximation are available as fitting formulae [14]. For the Rosseland opacities, the relevant fit has been produced in [15]. A correction factor to this fit was introduced in [16]; it is important at high ρ, where $\hbar\omega_{pe} \gtrsim k_B T$, and approximately takes into account the suppression of radiative transport at photon frequencies $\omega < \omega_{pe}$.

The radiative opacities in the atmosphere control the formation of the spectrum at a given radiation flux. This flux, however, is mainly determined by *conductive* opacities in subphotospheric layers. The most recent practical formulae for calculation of the conductive opacities in nonmagnetized, fully ionized plasmas, and relevant references can be found in [11, 17]. For the theory of neutron star thermal structure, thermal luminosity, and cooling, see the review [18] and references therein.

Partial ionization

A partially ionized plasma is present in neutron star envelopes at relatively low ρ and T. The larger the ion charge number Z, the higher the values of ρ and T up to which the plasma is partially ionized.

A theoretical description of the partial ionization can be based either on the physical picture or on the chemical picture of the plasma. In the chemical picture, bound objects (atoms, molecules, ions) are treated as elementary members of the thermodynamic ensemble, along with free electrons and nuclei. In the physical picture, nuclei and electrons (free and bound) are the only constituents of the ensemble. Relative merits and shortcomings of the two approaches are discussed, e.g., in [19], where many relevant references can be found. Models based on the chemical picture have the advantage that they provide detailed information about occupation numbers of various species in the plasma, which enables one to calculate radiative opacities.

In recent years, the nonmagnetic atmosphere models are usually based on the databases created due to two large-scale opacity projects: OP (Ref. [20], and references therein) and OPAL (e.g., [21]). The OP results are based entirely on the chemical picture, whereas the OPAL project uses the physical picture for the thermodynamic functions and introduces elements of the chemical picture for evaluating the abundances of species and calculating opacities. The OP database includes the most detailed atomic calculations. However, the OPAL data cover higher values of ρ, and therefore they are commonly used for modeling the dense atmospheres of neutron stars [22–24]. At still higher ρ, the nonideality effects lead to pressure ionization, which is hard to treat. For example, the equation of state for partially ionized carbon, which covers any densities at temperatures relevant for neutron stars ($T \sim 10^6$ K) and contains detailed information on occupation numbers, was developed only recently [25].

THE EFFECTS OF STRONG MAGNETIC FIELDS

Fully ionized plasma

Magnetic fields $B \gtrsim 10^{12}$ G, typical for isolated neutron stars, drastically modify many physical properties of matter (see, e.g., [26], for review). Motion of free electrons and ions perpendicular to the field lines is quantized into Landau orbitals with a characteristic transverse scale equal to the *magnetic length* $a_{\rm m} = (\hbar c/eB)^{1/2}$. This brings to the scene an atomic field-strength parameter $\gamma = (a_0/a_{\rm m})^2$. If this parameter is large, the Lorentz force acting on valence electrons in atoms exceeds the Coulomb force. The Landau energy levels of electrons are modified by relativistic effects if the field strength in the relativistic units, $b = \hbar\omega_{\rm c}/(m_e c^2) = B/B_{\rm r}$, is $\gtrsim 1$. Here, $\omega_{\rm c} = eB/(m_e c)$ is the electron cyclotron frequency ($\hbar\omega_{\rm c} \approx 11.577 B_{12}$ keV) and $B_{\rm r} = m_e^2 c^3/(e\hbar) = 4.414 \times 10^{13}$ G is the relativistic magnetic field unit. Introducing the notation $B_{12} = B/10^{12}$ G, we have $\gamma = 425.44 B_{12}$, and $b = \alpha_{\rm f}^2 \gamma = B_{12}/44.14$. A magnetic field is usually called *strong* if $\gamma \gg 1$ (typical for radio pulsars) and *superstrong* if $b \gg 1$ (which occurs in magnetars).

Magnetic field quantizes particle motion in Landau levels. In this case the decomposition of thermodynamic functions, like (2), remains useful, but its terms are modified. The quantization can be neglected if the particles occupy a great number of the Landau levels. For electrons, this happens if $\rho \gg \rho_B \approx 7045 B_{12}^{3/2} (\langle A \rangle / \langle Z \rangle)$ g cm^{-3} *or* if $k_B T \gg \hbar\omega_c$ (the latter criterion is approximate; see [1] for a more accurate one). In this case the thermodynamic functions of the plasma remain the same as at $B = 0$. In the opposite case, where $\rho < \rho_B$ *and* $k_B T \ll \hbar\omega_c$, the field is *strongly quantizing*: in equilibrium, all electrons reside in the ground Landau level. In the latter case, for example, the pressure of strongly degenerate ideal electron gas [cf. Eq. (3)] becomes $P_{id}^{(e)} \approx P_r b x_B^{\gamma_{ad}} / (2\pi^2 \gamma_{ad})$, where $x_B = (4\rho^2/3\rho_B^2)^{1/3} x_r$ is the magnetically modified relativity parameter ($x_B \propto \rho$), and γ_{ad} takes the values 3 and 2 in the non-relativistic ($x_B \ll 1$) and ultrarelativistic ($x_B \gg 1$) limits, respectively.

One can note that the *kinetic* pressure, calculated through electron velocities and momenta, is anisotropic in a magnetic field (e.g., [27, 28]). However, the kinetic pressure is only a part of the total pressure in the magnetized electron gas. Its anisotropy is exactly balanced by the pressure excess caused by magnetization currents [28, 29]. Thus, the thermodynamic pressure is isotropic (the stress tensor reduces to scalar) at any B.

Consecutive population of upper Landau levels by degenerate electrons with growing n_e leads to magnetic quantum oscillations of thermodynamic and kinetic functions. When the field is weakly quantizing, these quantities oscillate, as a rule, around their values obtained neglecting the magnetic quantization. For first-order (bulk) thermodynamic quantities (e.g., P), the oscillations are relatively weak, but for second-order quantities (e.g., r_e, heat capacity, or magnetic susceptibility [28]) they are more pronounced.

A magnetic field quantizes not only electrons, but also ions. Their Landau levels are separated by $\hbar\omega_{ci} \approx 6.35 (Z/A) B_{12}$ eV, where $\omega_{ci} = Z (m_e/m_i)\, \omega_c$ is the ion cyclotron frequency. In a plasma, this effect becomes appreciable (e.g., [1]) when $\omega_{ci} > \omega_{pi}$ *and* $\hbar\omega_{ci} > k_B T$. This happens at $B_{12} \gtrsim \max(100\sqrt{\rho_6}, 10\, T_6)$. In contrast to the case of electrons, the spin degeneracy of the Landau levels of the ions is taken off completely because of relatively large abnormal magnetic moments of the nuclei.

In the general case (arbitrary θ, x_r or x_B, Γ, and B), thermodynamic functions of electron-ion plasmas are given in [1] by analytical fitting functions of μ_e. A numerical inversion of $\rho(\mu_e)$ then provides all thermodynamic parameters as functions of ρ.

Under the conditions formulated in [30, 31], which are typical for strongly magnetized neutron star atmospheres, radiation propagates in the form of two *normal modes* with different elliptical polarizations. For so-called X-mode, the opacities are strongly suppressed by the field at $\omega \ll \omega_c$. This leads to suppression of the Rosseland opacities (fitted in [15]) and increase of the atmosphere densities. The thermal radiation from the atmosphere becomes polarized [32] and beamed [33] in a nontrivial way.

The two modes can partially convert to each other at certain ρ (for each ω) due to the *vacuum resonance*, which arises from interference of the plasma and vacuum polarizations [31]. This effect, which becomes very important in superstrong fields, has been recently studied in Ref. [34] (see also references therein).

The conductive opacities are also strongly affected by the magnetic field. First, the conduction becomes anisotropic, strongly suppressed in the direction transverse to the field. Second, the conductivities are modified by the quantizing magnetic field. A strongly quantizing field can change them by orders of magnitude; a weakly quantizing field makes them oscillating (like the oscillations of thermodynamic quantities, mentioned above). Fitting formulae for the conductive opacities due to electron-ion scattering have been derived in [35]. The impact of the magnetic fields on the thermal structure of neutron star envelopes is discussed, e.g., in Ref. [36] and references therein.

Partial ionization and condensed surface

Now let us consider *partially ionized and strongly magnetized* matter. In this case, one should take into account drastic changes of atoms and molecules, as well as formation of exotic species (e.g., molecular chains [37]). Moreover, the internal degrees of freedom of atoms, molecules, and ions become coupled to their center-of-mass motion. In addition, the strong magnetic field can induce formation of a condensed surface. Many authors attacked these problems during decades; a review can be found in [26]. The case of atomic hydrogen remains the only one, where atomic calculations have been completed with allowance for motion effects for any quantum states (bound [38] and continuum [39]), and presented as analytical fitting formulae [40]. Based on these results, thermodynamic functions of H plasmas have been calculated and tabulated for ρ, T, and B values typical for radio pulsars ($B \sim 10^{12} - 10^{13}$ G) [41, 42] and magnetars ($B \gtrsim 10^{14}$ G) [43]. These results were subsequently employed for calculation of radiative opacities and polarization vectors of normal electromagnetic modes in partially ionized, strongly magnetized H plasmas, and for modeling hydrogen atmospheres of neutron stars with strong magnetic fields – see [44] for review and references. Calculations of binding energies and oscillator strengths have been published also for the bound states of the He^+ ion [45, 46].

Apart from the one-electron systems H and He^+, there was no published calculations of quantum-mechanical properties of *moving* atoms and ions in strong magnetic fields, except for perturbational studies of restricted applicability ([47], and references therein). Recently Mori and Heyl [48] have implemented the perturbational approach in calculations of the ionization and dissociation equilibrium of the helium plasma.

The studies of the atoms, ions, and molecules, which *do not move* with respect to the field, have been much more numerous. Reviews can be found in [26] and in [1]. Especially note recent studies by Mori and Hailey [49] and Medin and Lai [50], who obtained the most credible binding energies for astrophysically important atoms, ions, and (in [50]) molecules, including the polymer chains, in strong magnetic fields. Mori and Hailey [49] calculated also oscillator strengths of bound-bound transitions. Mori and Ho [51] extended these calculations and used them for modeling strongly magnetized neutron star atmospheres composed of mid-Z elements (carbon, oxygen and neon). For bound-free transitions (photoionization), cross sections were calculated for H (with full account of the motion effects [39, 42, 43]) and He (with motion treated by perturbation [52]). Apart from H [41–43] and He [48], ionization equilibrium in strong magnetic fields at temperatures and densities relevant for neutron star atmospheres was calculated, although without full account of the motion effects, for C, O, Ne [51], and Fe [53].

Condensed surface. In the absence of the magnetic field, the temperature of neutron star atmosphere exceeds the critical temperature of a phase transition, so that a condensed zero-pressure surface is not formed. However, a strong magnetic field changes this situation. Ruderman [37] pointed out that polymer chains aligned with the strong magnetic fields should attract one another because of the quadrupole-quadrupole interactions, and eventually form a solid. The magnitude of such interaction for hydrogen chains was estimated by Lai and Salpeter [54] who concluded that hydrogen may form a solid stellar surface at superstrong fields ($B \gg 10^{13}$ G).

For iron and other heavy elements, the situation remained very uncertain until recently (see [26] for review). A great step forward, however, has been done by Medin and Lai [55], who calculated the electronic structure of one-dimensional infinite chains and three-dimensional condensed matter in strong magnetic fields ranging from $B = 10^{12}$ G to 2×10^{15} G using the density functional theory a local magnetic exchange-correlation function appropriate in the strong field regime, and taking into account the electron band structure. They computed the work function and studied the cohesive property of three-dimensional condensed matter of H, He, C, and Fe at zero pressure, constructed from interacting chains in a body-centered tetragonal lattice. Such three-dimensional condensed matter is found to be bound relative to individual atoms, with the cohesive energy increasing rapidly with increasing B. Reflection and emission properties of a condensed matter in strong magnetic fields and spectra and polarization of thermal radiation emitted by such surface were studied in [56] (see also references therein).

CONCLUDING REMARKS

Surface layers of neutron stars possess a number of unusual properties due to the high gravity (and therefore high densities of the envelopes) and strong magnetic fields. In this brief review I have given only a little slice of the problems and developments in the theory of neutron star envelopes. However, even this slice demonstrates the immense richness of physical phenomena which are unique to neutron star envelopes, especially those with strong and superstrong magnetic fields. Many of these phenomena and properties require further study.

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REFERENCES

1. P. Haensel, A. Y. Potekhin, and D. G. Yakovlev, *Neutron Stars. 1. Equation of State and Structure*, vol. 326 of *Astrophys. Space Sci. Library*, Springer, New York, 2007.
2. O. Y. Gnedin, D. G. Yakovlev, and A. Y. Potekhin, *Mon. Not. Roy. astron. Soc.* **324**, 725–736 (2001).
3. A. D. Kaminker, M. E. Gusakov, D. G. Yakovlev, and O. Y. Gnedin, *Mon. Not. Roy. astron. Soc.* **365**, 1300–1308 (2006).
4. A. I. Chugunov, and D. G. Yakovlev, *Astronomy Reports* **49**, 724–738 (2005).
5. M. B. Larson, and B. Link, *Mon. Not. Roy. astron. Soc.* **333**, 613–622 (2002).
6. C. J. Pethick, and D. G. Ravenhall, *Annu. Rev. Nucl. Part. Sci.* **45**, 429–484 (1995).

7. J. P. Hansen, G. M. Torrie, and P. Vieillefosse, *Phys. Rev. A* **16**, 2153–2168 (1977).
8. H. DeWitt, W. Slattery, and G. Chabrier, *Physica B: Condensed Matter* **228**, 21–26 (1996).
9. D. K. Nadyozhin, and A. V. Yudin, *Astronomy Letters* **31**, 271–279 (2005).
10. D. G. Yakovlev, *Astrophys. Space Sci.* **98**, 37–59 (1984).
11. A. Y. Potekhin, D. A. Baiko, P. Haensel, and D. G. Yakovlev, *Astron. Astrophys.* **346**, 345–353 (1999).
12. A. Y. Potekhin, and G. Chabrier, *Phys. Rev. E* **62**, 8554–8563 (2000).
13. G. Chabrier, *Astrophys. J.* **414**, 695–700 (1993).
14. D. G. Hummer, *Astrophys. J.* **327**, 477–484 (1988).
15. A. Y. Potekhin, and D. G. Yakovlev, *Astron. Astrophys.* **374**, 213–226 (2001).
16. A. Y. Potekhin, D. G. Yakovlev, G. Chabrier, and O. Y. Gnedin, *Astrophys. J.* **594**, 404–418 (2003).
17. S. Cassisi, A. Y. Potekhin, A. Pietrinferni, M. Catelan, and M. Salaris, *Astrophys. J.* **661**, 1094–1104 (2007).
18. D. G. Yakovlev, and C. J. Pethick, *Annu. Rev. Astron. Astrophys.* **42**, 169–210 (2004).
19. F. J. Rogers, and C. A. Iglesias, *Space Science Reviews* **85**, 61–70 (1998).
20. C. Mendoza, M. J. Seaton, P. Buerger, et al., *Mon. Not. Roy. astron. Soc.* **378**, 1031–1035 (2007).
21. C. A. Iglesias, and F. J. Rogers, *Astrophys. J.* **464**, 943–+ (1996).
22. V. E. Zavlin, G. G. Pavlov, and Y. A. Shibanov, *Astron. Astrophys.* **315**, 141–152 (1996).
23. M. Rajagopal, and R. W. Romani, *Astrophys. J.* **461**, 327–333 (1996).
24. B. T. Gänsicke, T. M. Braje, and R. W. Romani, *Astron. Astrophys.* **386**, 1001–1008 (2002).
25. A. Y. Potekhin, G. Massacrier, and G. Chabrier, *Phys. Rev. E* **72**, 046402 (2005).
26. D. Lai, *Rev. Mod. Phys.* **73**, 629–662 (2001).
27. V. Canuto, and H.-Y. Chiu, *Phys. Rev.* **173**, 1210–1219 (1968).
28. R. D. Blandford, and L. Hernquist, *J. Phys. C: Solid State Phys.* **15**, 6233–6243 (1982).
29. V. Canuto, *Phys. Rev. A* **3**, 648–650 (1971).
30. Y. N. Gnedin, and G. G. Pavlov, *Sov. Phys. – JETP* **38**, 903 (1974).
31. W. C. G. Ho, and D. Lai, *Mon. Not. Roy. astron. Soc.* **338**, 233–252 (2003).
32. G. G. Pavlov, and V. E. Zavlin, *Astrophys. J.* **529**, 1011–1018 (2000).
33. G. G. Pavlov, Y. A. Shibanov, J. Ventura, and V. E. Zavlin, *Astron. Astrophys.* **289**, 837–845 (1994).
34. M. van Adelsberg, and D. Lai, *Mon. Not. Roy. astron. Soc.* **373**, 1495–1522 (2006).
35. A. Y. Potekhin, *Astron. Astrophys.* **351**, 787–797 (1999).
36. A. Y. Potekhin, G. Chabrier, and D. G. Yakovlev, *Astrophys. Space Sci.* **308**, 353–361 (2007).
37. M. Ruderman, *Phys. Rev. Lett.* **27**, 1306–1308 (1971).
38. A. Y. Potekhin, *J. Phys. B: At. Mol. Opt. Phys.* **27**, 1073–1090 (1994).
39. A. Y. Potekhin, and G. G. Pavlov, *Astrophys. J.* **483**, 414–425 (1997).
40. A. Y. Potekhin, *J. Phys. B: At. Mol. Opt. Phys.* **31**, 49–63 (1998).
41. A. Y. Potekhin, G. Chabrier, and Y. A. Shibanov, *Phys. Rev. E* **60**, 2193–2208 (1999).
42. A. Y. Potekhin, and G. Chabrier, *Astrophys. J.* **585**, 955–974 (2003).
43. A. Y. Potekhin, and G. Chabrier, *Astrophys. J.* **600**, 317–323 (2004).
44. A. Y. Potekhin, G. Chabrier, D. Lai, W. C. G. Ho, and M. van Adelsberg, *J. Phys. A: Math. Gen.* **39**, 4453–4458 (2006).
45. V. G. Bezchastnov, G. G. Pavlov, and J. Ventura, *Phys. Rev. A* **58**, 180–185 (1998).
46. G. G. Pavlov, and V. G. Bezchastnov, *Astrophys. J.* **635**, L61–L64 (2005).
47. G. G. Pavlov, and P. Mészáros, *Astrophys. J.* **416**, 752–761 (1993).
48. K. Mori, and J. S. Heyl, *Mon. Not. Roy. astron. Soc.* **376**, 895–906 (2007).
49. K. Mori, and C. J. Hailey, *Astrophys. J.* **564**, 914–929 (2002).
50. Z. Medin, and D. Lai, *Phys. Rev. A* **74**, 062507 (2006).
51. K. Mori, and W. C. G. Ho, *Mon. Not. Roy. astron. Soc.* **377**, 905–919 (2007).
52. Z. Medin, D. Lai, and A. Y. Potekhin, *Mon. Not. Roy. astron. Soc.* (submitted, 2007), `arXiv:0704.1598`.
53. M. Rajagopal, R. W. Romani, and M. C. Miller, *Astrophys. J.* **479**, 347–356 (1997).
54. D. Lai, and E. E. Salpeter, *Astrophys. J.* **491**, 270–285 (1997).
55. Z. Medin, and D. Lai, *Phys. Rev. A* **74**, 062508 (2006).
56. M. van Adelsberg, D. Lai, A. Y. Potekhin, and P. Arras, *Astrophys. J.* **628**, 902–913 (2005).