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Generation of magnetosonic waves and formation of structures in galaxies

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ABSTRACT

We study the generation of magnetosonic waves in galactic gaseous discs taking account of the magnetic field, differential rotation and self-gravity. The special case of perturbations is considered with the wavevector perpendicular to the magnetic field. The necessary condition of the amplification of seed perturbations is the presence of differential rotation and non-vanishing radial component of the magnetic field that can easily be satisfied in galactic discs. Differential rotation stretches the azimuthal field from the radial one and, therefore, we consider the generation of waves on the time-dependent background magnetic field. Basically, an amplification is rather efficient, and seed perturbations become non-linear already after several rotation periods for a wide range of wavelength. The generated magnetosonic waves can be either non-oscillatory or oscillatory depending on the parameters of gas. If perturbations are Jeans stable, then typically non-oscillatory waves are amplified. However, interplay between self-gravity, magnetic field and rotational shear can change qualitatively the classical Jeans instability, so that the latter becomes oscillatory and tends to be suppressed in galaxies.

Key words: instabilities – MHD – cosmic rays – galaxies: ISM – galaxies: magnetic fields.

1 INTRODUCTION

The interstellar medium (ISM) consists of many different phases with a wide variety of properties. Partly, these phases can be generated by different types of instability which occur in ISM conditions (see discussion in Kim, Ostriker & Stone 2002 for more details). Instabilities should coherently amass material over length-scales much longer than the atomic mean-free path, therefore, they can be considered in a magnetohydrodynamic approximation. In galaxies, one can distinguish global and local instabilities. The first ones are characterized by length-scales comparable to the galactic radius whereas the second ones occur at a much shorter length-scale. Most likely, the best known example of global instability is the one responsible for the formation of spiral arms. Recently, a detailed study of such instability has been done by Gomez & Cox (2002a,b) and Gomez & Cox (2004) who performed 3D magnetohydrodynamic (MHD) simulations using a realistic galactic model which includes the dynamic effects of disc, disc halo and bulge. The authors impose the seed non-axisymmetric density waves and study the gas response to such perturbations over the dominant gravitational axisymmetric galactic components. They concluded that the density waves can reach of a reasonable magnitude after ∼800 Myr. The authors also analysed the effect of the azimuthal magnetic field and found that it can lead to variability of the structure above the plane with the characteristic time-scale ∼60 Myr. The evolution of spiral disturbances under the combined influence of gravitational and magnetorotational instabilities has been considered recently by Fromang et al. (2004). They argued that the magnetorotational instability leads to turbulence and lowers the strength of the gravitational stress tensor. This tensor exhibits periodic oscillations which are not present in hydrodynamic simulations. They attribute such a behaviour to the presence of a second spiral mode that can be excited by the high-frequency motions associated with turbulence.

In contrast to global instabilities, the local ones occur on shorter length-scales and, considering such instabilities, one can assume that the large-scale structure of a galaxy is fixed. One of the examples of such local instabilities in galaxies is the Parker instability (Paker 1966) that generally can be relevant to the formation of some structures in galaxies (e.g. Mouschovias 1974; Shibata & Matsumoto 1991). On relatively large scales, the Parker instability has to be accompanied by the self-gravity of perturbations (the Jeans instability) and, perhaps, the combined Parker–Jeans instability can lead to the formation of filament-like structures in the ISM. Note, however, that the study of this type of instability in 3D shows that, most likely, it produces small-scale structures rather than the giant molecular clouds (Asseo et al. 1978). Numerical simulations in 3D also indicate that the Parker instability alone is unable to produce structures like giant molecular clouds or associations (Basu, Mouschovias & Paleologou 1997; Kim et al. 1998; Kim, Ryu & Jones 2001). Note also that a random component of the galactic magnetic field should suppress the Parker instability significantly

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(Kim & Ryu 2001). Therefore, it cannot be excluded that the role of the Parker instability is often exaggerated, and other instabilities play an important role in formation of large-scale structures in galaxies.

One more destabilizing effect can be caused by rotation which is essentially differential in galaxies. For example, differential rotation can change the properties of the Parker and Parker–Jeans instabilities. Hanasz & Lesch (1997) and Hanasz (1997) consider the Parker instability of flux tubes in galactic discs taking account of cosmic rays and differential rotation. They calculated the dynamo coefficients associated with this instability and concluded that the dynamo α-parameter is significantly magnified in the arms and diminished in the inter-arm regions due to the influence of cosmic rays and differential rotation. Apart from that differential rotation of a magnetized galactic gas should initiate some new instabilities which can play an important role in the formation of large ISM clouds. Recently, a study of this sort of effects has been undertaken by Kim, Ostriker & Stone (2002) and Kim, Ostriker & Stone (2003). The authors considered stability of the galactic discs taking into account the magnetorotational effects and adopting simplified initial magnetic configurations (either toroidal or vertical magnetic fields). They found that MHD-phenomena such as the magneto-Jeans instability (which is the sort of non-axisymmetric Jeans instability in differentially rotating disc with embedded azimuthal magnetic field, see Kim & Ostriker 2001, 2002; Kim et al. 2002) and magnetorotational instability can appreciably contribute to the formation of large-scale clouds. Kim et al. (2002) argued that the magneto-Jeans instability is likely to be one of the most powerful cloud-condensing mechanism which may occur in spiral arms or galactic centres.

The above examples illustrate the importance of MHD instabilities for dynamics of the ISM. In this paper, we consider one additional MHD instability that can occur in ISM conditions and lead to the formation of structures in galaxies. We show that the magnetorotational effects are sensitive to the geometry of the magnetic field, and the stability properties of galaxies can be more complicated compared to those considered by Kim et al. (2002) and Kim et al. (2003) if the magnetic geometry is more complex. The effect treated in this paper is relevant to differential rotation and the presence of a non-vanishing radial component of the magnetic field. This type of instability has already been considered by Bonanno & Urpin (2006, 2007a) and Bonanno & Urpin (2007b) for different conditions. In this paper, we show that the magnetic shear-driven instability can operate in the galactic environment even in the process of generation of the toroidal field from the radial one when the background magnetic field depends on time. The main goal of our study is a treatment of the physical effect that can be important in galaxies rather than calculations of a consistent galactic model. The study of the new instability presented in this paper can be useful in the interpretation of numerical calculations of the galactic models and in understanding the nature of galactic structures.

The paper is organized as follows. In Section 2, we discuss basic equations governing the shear-driven instability in galaxies. In Section 3, the results of numerical calculations are presented. A summary of the results and their possible applications to galaxies is given in Section 4.

2 BASIC EQUATIONS

We consider MHD processes in a galaxy using a two-fluid approximation and assuming that the ISM consists of thermal gas and cosmic rays. The latter is treated as gas with a significant pressure but with negligible density. It is assumed that there is no energy exchange between the fluids, and they interact only dynamically via partial pressures. The evolution of cosmic rays can be described in the diffusion approximation taking into account only the diffusion along magnetic field lines because the gas of cosmic rays is strongly magnetized and diffusion across the field line is usually unimportant (see e.g. Drury & Völk 1981; Ko 1992; Giacalone & Jokipii 1999). The MHD equations, governing the coupled evolution of the thermal gas, magnetic and gravitational fields, and cosmic rays read in this approximation:

$$\dot{\boldsymbol{v}} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\frac{\nabla P}{\rho} - \nabla\psi + \frac{1}{4\pi\rho}(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}, \tag{1}$$

$$\dot{\rho} + \nabla \cdot (\rho \boldsymbol{v}) = 0, \tag{2}$$

$$\dot{P}_{\rm g} + \boldsymbol{v} \cdot \nabla P_{\rm g} + \gamma_{\rm g} P_{\rm g} \nabla \cdot \boldsymbol{v} = 0, \tag{3}$$

$$\Delta\psi = 4\pi G\rho, \tag{4}$$

$$\dot{\boldsymbol{B}} - \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) = 0, \tag{5}$$

$$\nabla \cdot \boldsymbol{B} = 0, \tag{6}$$

$$\dot{P}_{\rm c} + \boldsymbol{v} \cdot \nabla P_{\rm c} + \gamma_{\rm c} P_{\rm c} \nabla \cdot \boldsymbol{v} = \nabla \cdot (\kappa_{\parallel} \nabla_{\parallel} P_{\rm c}). \tag{7}$$

We complemented the standard set of MHD equations with the inclusion of the diffusion-convection equation for cosmic rays (see e.g. Ryu et al. 2003) and the Poisson equation for the gravitational potential. Our notation is as follows. ρ and $\boldsymbol{v}$ are the thermal gas density and velocity, respectively; $P = P_{\rm g} + P_{\rm c}$ is the total (gas plus cosmic rays) pressure; $\boldsymbol{B}$ is the magnetic field, ψ is the gravitational potential, $\kappa_{\parallel}$ is the diffusion coefficient of cosmic rays along the magnetic field; $\nabla_{\parallel} P_{\rm c} = \boldsymbol{B}(\boldsymbol{B} \cdot \nabla P_{\rm c})/B^2$; $\gamma_{\rm g}$ and $\gamma_{\rm c}$ are the adiabatic indexes of the gas and cosmic rays. The adiabatic index of the cosmic rays is defined as $\gamma_{\rm c} = 1 + P_{\rm c}/E_{\rm c}$ where $E_{\rm c}$ is the particle energy per unit volume. This index can be simply related to the form of the cosmic ray momentum distribution if the latter is a power law (see e.g. Ryu et al. 2003). In particular, a momentum distribution $\propto p^{-q}$ with the index between 4 and 5 appropriate for galactic cosmic rays (see e.g. Blandford & Eichler 1987) lead to $P_{\rm c}/E_{\rm c} = (q-3)/3 \approx (1-2)/3$ and hence, $\gamma_{\rm c} \approx (4\text{–}5)/3$. This model is often used with $q = 14/3$, then $\gamma_{\rm c} = 14/9$. For the sake of simplicity, we consider in this paper a polytropic gas and assume $P_{\rm g} = c_{\rm g}^2 \rho$ where $c_{\rm g}$ is the sound speed.

We work in cylindrical coordinates $(s\ \varphi, z)$ with the unit vectors $(\boldsymbol{e}_s, \boldsymbol{e}_\varphi, \boldsymbol{e}_z)$. The basic state on which the stability analysis is performed is assumed to be axisymmetric with the angular velocity $\Omega = \Omega(s)$ and $\boldsymbol{B} \neq 0$. In the presence of non-vanishing radial field $B_{\rm s}$ and differential rotation, the azimuthal field increases with time by winding up the radial field lines. If the magnetic Reynolds number is large, then one obtains from equation (5) that the azimuthal field grows linearly with time in the basic state,

$$B_\varphi(t) = B_\varphi(0) + s\Omega' B_{\rm s} t, \tag{8}$$

where $B_\varphi(0)$ is the azimuthal field at $t = 0$. A growth of B_φ given by equation (8) can last only while diffusion of the toroidal field is negligible. Eventually, in the presence of a finite diffusivity, a steady state will emerge where winding up is balanced by diffusion of the azimuthal field. The time-scale to reach this steady state is approximately equal to the diffusion time-scale, $\sim s^2/\eta$, where η is the magnetic diffusivity. Therefore, we restrict our consideration by $t < s^2/\eta$ when equation (8) is valid.

In the unperturbed state, the system is assumed to be in hydrostatic equilibrium in the s- and z-directions,

$$\frac{\nabla P}{\rho} = \boldsymbol{D} + \frac{1}{4\pi\rho}(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}, \quad \boldsymbol{D} = -\nabla\psi + \Omega^2 \boldsymbol{s}. \tag{9}$$

Note that, generally, this condition is rather difficult to satisfy, and hydrostatic equilibrium cannot be reached for an arbitrary magnetic field.

To demonstrate a possibility of the magnetic shear-driven instability in the galactic environment, we consider a very simplified model of the baseground magnetic field assuming that $B_s \propto B_\varphi(0) \propto B_\varphi(t) \propto 1/s$ and $B_z = $ constant. Then, $\nabla \times \boldsymbol{B}(t) = 0$ and the Lorentz force is vanishing in equation (9). If, additionally, gravity is approximately radial, $\boldsymbol{g}(s) = -g(s)\boldsymbol{e}_s$, and the centrifugal force is balanced by gravity, $g(s) = s\Omega^2$, then the pressure and, hence, density can be treated as homogeneous in the basic state. As it follows from equation (8), the radial dependence of the toroidal field remains unchanged in this model if $s\Omega' = $ const, or $\Omega = A \ln(s/s_1) + \Omega(s_1)$ where A is constant and $\Omega(s_1)$ is the value of the angular velocity at $s = s_1$. This dependence differs from the galactic differential rotation for which one has $\Omega \propto 1/s$ and $q = s\Omega'/\Omega = -1$. Nevertheless, since we consider only a local instability arising at some radius s_1, we can always choose constant A in such a way that the angular velocity Ω and rotational shear q are approximately equal to their galactic values in the neighborhood of a cylindrical radius s_1. Supposing $A = -\Omega(s_1)$, we have

$$\Omega(s) = \Omega(s_1)\,[1 - \ln(s/s_1)]\ , \quad q = -\,[1 - \ln(s/s_1)]^{-1}\ . \tag{10}$$

If we choose now $\Omega(s_1)$ equal to the galactic angular velocity at the radius s_1, then both the angular velocity and rotational shear will agree with the galactic values in the neighborhood of s_1. The same concerns also gravity $g(s)$. Integrating Poisson equation (4) and assuming that ρ is homogeneous, we obtain that $g = 2\pi G\rho s + D/s$ in the equilibrium state where D is constant. Again, this gravity differs from the galactic gravity. However, since we consider only local processes in the neighbourhood of some point s_1, there is no need to obtain the expression for g that is valid everywhere in the galaxy. We can choose the value of constant D in such a way that hydrostatic equilibrium will be satisfied in the neighbourhood of s_1 with the accuracy in linear terms and gravity will be equal to its galactic value at $s = s_1$. Therefore, our model qualitatively can mimic galactic differential rotation and gravity for the consideration of local instabilities.

As it was mentioned, instabilities can play an important role in the formation of different galactic structures with various length-scales. In this paper, we consider linear stability and determine the condition at which instability occurs, and its growth rate. Within this approach, all quantities can be represented as the sum of an unperturbed part that characterizes the basic state and a small perturbation. Since we study the instability that could lead to formation of galactic structures, it has to be assumed that there are no structures in the basic state, and these structures will be formed as a result of the development of seed perturbations. The non-linear evolution is a much more complicated problem and it will be considered elsewhere.

Consider stability of axisymmetric short wavelength perturbations with the spatial dependence $\propto \exp(-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r})$, where $\boldsymbol{k} = (k_s, 0, k_z$ is the wavevector, $|\boldsymbol{k}\cdot\boldsymbol{r}| \gg 1$. Since we consider a short wavelength approximation, all unperturbed quantities including gravity can be treated as constant. The advantage of this approximation is that it does not depend on the boundary conditions, and the results can be applied to any galactic model if the wavelength of perturbations is shorter than the length-scale of unperturbed quantities. Small perturbations will be indicated by subscript 1, while unperturbed quantities will have no subscript, except for indicating vector components. Then, the linearized MHD equations read with accuracy in terms of the lowest order in $|\boldsymbol{k}\cdot\boldsymbol{r}|^{-1}$

$$\frac{\partial \boldsymbol{v}_1}{\partial t} + 2\boldsymbol{\Omega} \times \boldsymbol{v}_1 + \boldsymbol{e}_\varphi s\Omega' v_{1s} = \frac{\mathrm{i}\,\boldsymbol{k}P_1}{\rho} + \mathrm{i}\boldsymbol{k}\psi_1 - \frac{\mathrm{i}}{4\pi\rho}(\boldsymbol{k}\times\boldsymbol{B}_1)\times\boldsymbol{B}), \tag{11}$$

$$\frac{\partial \rho_1}{\partial t} - \mathrm{i}\rho(\boldsymbol{k}\cdot\boldsymbol{v}_1) = 0, \tag{12}$$

$$\frac{\partial P_{g1}}{\partial t} - \mathrm{i}\gamma_g P_g(\boldsymbol{k}\cdot\boldsymbol{v}_1) = 0, \tag{13}$$

$$k^2\psi_1 = -4\pi G\rho_1, \tag{14}$$

$$\frac{\partial \boldsymbol{B}_1}{\partial t} = \boldsymbol{e}_\varphi s\Omega' B_{1s} - \mathrm{i}(\boldsymbol{B}\cdot\boldsymbol{k})\boldsymbol{v}_1 + \mathrm{i}\boldsymbol{B}(\boldsymbol{k}\cdot\boldsymbol{v}_1), \tag{15}$$

$$\boldsymbol{k}\cdot\boldsymbol{B}_1 = 0, \tag{16}$$

$$\left(\frac{\partial}{\partial t} + \omega_{cr}\right) P_{c1} - \mathrm{i}\gamma_c P_c(\boldsymbol{k}\cdot\boldsymbol{v}_1) = 0, \tag{17}$$

where the inverse time-scales of cosmic rays diffusion are given by $\omega_{cr} = \kappa_\parallel(\boldsymbol{k}\cdot\boldsymbol{B})^2/B^2$. In these equations, we take into account the gradient term associated with the disc differential rotation since the terms proportional to $s\Omega'$ and Ω in equation (11) are of the same order of magnitude. A non-uniformity of all other quantities characterizing the basic state is neglected in a short wavelength approximation.

In this paper, we treat only a special type of perturbations with $\boldsymbol{k}\cdot\boldsymbol{B} = 0$, but the results are qualitatively same for any other perturbations. This choice of $\boldsymbol{k}$ is made by simplicity reasons rather than by physical motivation because the basic equations can be simplified substantially for such perturbations. Nevertheless, the case $\boldsymbol{k}\cdot\boldsymbol{B} = 0$ illustrates well the main qualitative features of the shear-driven instability in the time-dependent basic state. Moreover, the standard magnetorotational instability does not operate in this case because its growth rate is proportional to $\boldsymbol{k}\cdot\boldsymbol{B}$.

By combining equations (11)–(17), we obtain after some algebra the following equation for $f_1 = (\boldsymbol{k}\cdot\boldsymbol{v}_1)$

$$\frac{\mathrm{d}^4 f_1}{\mathrm{d}t^4} + k^2\left[c_G^2 + c_{As}^2\frac{k^2}{k_z^2} + c_{A\varphi}^2(t)\right]\frac{\mathrm{d}^2 f_1}{\mathrm{d}t^2} + 6\omega_{B\Omega}^3\frac{\mathrm{d}f_1}{\mathrm{d}t} + 6k^2c_{As}^2(s\Omega')^2 f_1 = -\kappa^2 k_s\frac{\mathrm{d}^2 v_{1s}}{\mathrm{d}t^2}, \tag{18}$$

where

$$c_G^2 = c_g^2 + c_c^2 - 4\pi G\rho/k^2, \quad c_g = \sqrt{\frac{\gamma_g P_g}{\rho}}, \ c_c = \sqrt{\frac{\gamma_c P_c}{\rho}},$$

$$c_{As} = \frac{B_s}{\sqrt{4\pi\rho}}\ , \quad c_{A\varphi}(t) = \frac{B_\varphi(t)}{\sqrt{4\pi\rho}}\ , \quad \kappa^2 = 2\Omega(2\Omega + s\Omega')\ ,$$

$$\omega_{B\Omega}^3 = k^2 c_{A\varphi}(t)c_{As}s\Omega'.$$

If $k_s = 0$ and the pressure of cosmic rays and self-gravity are negligible, then equation (12) of the paper by Bonanno & Urpin (2007b) can be recovered from equation (18) if one replaces c_G^2 by the square of the sound speed c_s^2. However, this difference can be important in the galactic environment because, in the contrast to

c_s^2, the quantity c_G^2 is negative for Jeans unstable perturbations with $k^2 < 4\pi G\rho/(c_g^2 + c_c^2)$.

In equation (18), the ratio of the term on the right-hand side and the second term on the left-hand side is of the order of $\Omega^2/k^2 s_G^2 \sim \Omega^2/k^2 c_g^2$ if perturbations are not marginally stable to the Jeans instability ($c_G^2 \approx 0$). Since $\Omega \sim c_g/H$ in the galactic disc where H is the disc thickness, we obtain that the ratio of these two terms is $\sim(kH)^{-2}$, and the term on the right-hand side should be neglected in a short wavelength approximation. Then, we have

$$\frac{\mathrm{d}^4 f_1}{\mathrm{d}t^4} + k^2 \left[c_G^2 + c_{As}^2 \frac{k^2}{k_z^2} + c_{A\varphi}^2(t) \right] \frac{\mathrm{d}^2 f_1}{\mathrm{d}t^2} + 6\omega_{B\Omega}^3 \frac{\mathrm{d} f_1}{\mathrm{d}t} + 6k^2 c_{As}^2 (s\Omega')^2 f_1 = 0. \quad (19)$$

Equation (19) with the corresponding initial conditions describes the behaviour of velocity perturbations during the initial stage of evolution when differential rotation stretches toroidal field lines from the poloidal ones, and the toroidal field still has not reached saturation. If we suppose either $\Omega' = 0$ or $B_s = 0$, then equation (20) simplifies to

$$\frac{\mathrm{d}^4 f_1}{\mathrm{d}t^4} + k^2 \left(c_G^2 + c_{A\varphi 0}^2 \right) \frac{\mathrm{d}^2 f_1}{\mathrm{d}t^2} = 0, \quad (20)$$

where $c_{A\varphi 0} = B_\varphi(0)/\sqrt{4\pi\rho}$. This equation has two different types of solutions. If $c_G^2 + c_{A\varphi 0}^2 > 0$, then equation (20) has an oscillatory solution that corresponds to cylindrical magnetosonic waves modified by gravity. The frequency of such waves is $k\sqrt{c_G^2 + c_{A\varphi 0}^2}$. On the contrary, if $c_G^2 + c_{A\varphi 0}^2 < 0$, the solution is aperiodic and describes the Jeans instability modified by the presence of the toroidal field. In this paper, we consider the behaviour of perturbations under the combined influence of Ω' and B_s, when perturbations are governed by equation (19), and will show that solutions can be qualitatively different.

Having calculated f_1, the behaviour of perturbations of the density, radial and azimuthal magnetic fields can be obtained then from equations (12) and (15), respectively.

3 NUMERICAL RESULTS

To follow the behaviour of perturbations, it is convenient to introduce dimensionless quantities

$$\tau = \Omega t\ ,\quad x = kH\ ,\quad H = \frac{\sqrt{c_g^2 + c_c^2}}{\Omega}\ ,\quad \mu = \frac{k_z^2}{k^2}\ ,\quad \varepsilon = \frac{4\pi G\rho}{\Omega^2},$$
$$\delta_s = \frac{c_{As}^2}{c_g^2 + c_c^2}\ ,\quad \delta_\varphi = \frac{c_{A\varphi 0}^2}{c_g^2 + c_c^2}\ ,\quad q = \frac{s\Omega'}{\Omega}.$$

Then, introducing $f = f_1(t)/f_1(0)$ where $f_1(0)$ is the initial value of $f_1 = (\boldsymbol{k} \cdot \boldsymbol{v}_1)$, we obtain from equation (19)

$$\frac{\mathrm{d}^4 f}{\mathrm{d}\tau^4} + x^2 \left[1 + \frac{\delta_s}{\mu} + \left(\sqrt{\delta_\varphi} + q\sqrt{\delta_s}\tau \right)^2 - \frac{\varepsilon}{x^2} \right] \frac{\mathrm{d}^2 f}{\mathrm{d}\tau^2} + 6qx^2\sqrt{\delta_s}\left(\sqrt{\delta_\varphi} + q\sqrt{\delta_s}\tau \right) \frac{\mathrm{d}f}{\mathrm{d}\tau} + 6q^2x^2\delta_s f = 0. \quad (21)$$

The dependence of the solution on the wavelength is characterized by the parameter x and, on the initial magnetization of gas, by the parameters δ_s and δ_φ. To solve equation (21), one needs the initial conditions for three time derivatives of f. In calculations, we try different initial conditions because their choice is determined by the origin of perturbations which are uncertain.

Equations (12) and (15) can be written in a dimensionless form as

$$\frac{d}{\mathrm{d}\tau}\left(\frac{\rho_1}{\rho}\right) = \mathrm{i} f \xi, \quad (22)$$

$$\frac{d}{\mathrm{d}\tau}\left(\frac{B_{1s}}{B_s}\right) = \mathrm{i} f \xi, \quad (23)$$

$$\frac{d}{\mathrm{d}\tau}\left(\frac{B_{1\varphi}}{B_s}\right) = q\left(\frac{B_{1s}}{B_s}\right) + \mathrm{i} f \xi \left(\sqrt{\frac{\delta_\varphi}{\delta_s}} + q\tau \right), \quad (24)$$

where

$$\xi = \frac{H[\boldsymbol{k} \cdot \boldsymbol{v}_1(0)]}{\sqrt{c_g^2 + c_c^2}}.$$

The parameter ξ characterizes the initial compressibility of the galactic gas. For the sake of simplicity, we assume in calculations that the initial perturbations of the magnetic field are vanishing, $B_{1s}(0) = B_{1\varphi}(0) = 0$, and the initial value of ρ_1/ρ is 0.1. Equations (21)–(24) were solved numerically for a wide range of the parameters.

To mimic the galactic rotation, we suppose $q = -1$ in equation (21). Since the age of the galaxy is approximately $40P$ where P is the rotation period, we do not need to study a long-term behaviour of perturbations but restrict ourselves by $t < 40\,P$ that corresponds to the dimensionless time $\tau < 250$. The long-term behaviour in a time-dependent basic state has been considered in detail by Bonanno & Urpin (2007b). The compressibility parameter ξ can be estimated as $\sim(kH)\ v_1(0)/c_g$ where $kH > 1$ in a short wavelength approximation. Assuming that the initial velocity perturbation is a small fraction of the sound speed, we can estimate $\xi \sim 0.01$–1.

In Fig. 1, we plot the time dependence of the perturbations of compressibility f, normalized density and the azimuthal magnetic field for $x = 20$, $\mu = 0.5$, $\varepsilon = 0.1$ and relatively weak unperturbed magnetic field with $\delta_s = \delta_\varphi = 0.001$. Such perturbations are almost not influenced by self-gravity and, hence, are Jeans stable. We assume that $\mathrm{d}f/\mathrm{d}\tau = 1$ and $\mathrm{d}^2 f/\mathrm{d}t^2 = \mathrm{d}^3 f/\mathrm{d}t^3 = 0$ at $\tau = 0$. Note that in all considered cases, the behaviour of the normalized radial

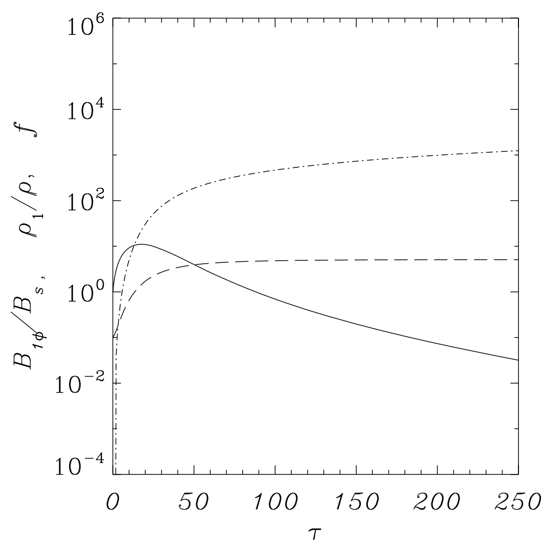

Figure 1. The time dependence of f (solid line), $|\rho_1/\rho|$ (dashed line) and $|B_{\varphi 1}/B_s|$ (dash–dotted line) for $\delta_s = \delta_\varphi = 0.001$, $x = 20$, $\mu = 0.5$, $\xi = 0.01$ and $\varepsilon = 0.1$.

magnetic field B_{1s}/B_s and density ρ_1/ρ is practically the same because they are governed by the same equations (22) and (23). The minor difference is only because of our choice of initial conditions for these quantities but this difference cannot be seen in figures. That is why we plot only ρ_1/ρ in figures. The compressibility f reaches its maximum rather rapidly and becomes a factor of $\approx$10 greater than its initial value after $\tau \approx 20$ that corresponds to three to four rotation periods. Then, it gradually decreases with the characteristic time-scale $\sim$10–20P. However, despite the decrease in f, perturbations of the density and magnetic field continue to grow because they are determined by f integrated over time. The density perturbations become comparable to ρ already after $\approx$3–4 revolutions. At the same time, perturbations of the azimuthal field are greater than the initial field by a factor of $\sim$10. At the late stage, $\tau \sim 200$, the amplitude of magnetic perturbations is of the order of unperturbed quantities, and our linear analysis does not apply. It is worth noting that the results are not very sensitive to the value x. We made a run for the same parameters and $x = 5$, and the difference with Fig. 1 is less than 10 per cent.

In Fig. 2, we show the same dependences but for the case when the compressibility parameter is larger, $\xi = 0.1$. Since equation (21) does not depend on ξ, the evolution of f does not differ from that shown in Fig. 1. On the contrary, perturbations of the density and magnetic field are approximately of the order of magnitude greater. Therefore, for such value of the compressibility parameter, perturbations can grow significantly during the very early evolutionary stage and, most likely, reach a non-linear regime after $\sim$10–20 rotation periods.

To demonstrate the dependence on the shape of perturbations, we plot in Fig. 3 the evolution of the same quantities as in Figs 1 and 2 but for $\mu = 0.0005$. This μ corresponds to filament-like structures with a very short radial wavelength and the ratio of radial and vertical wavelengths $\approx$0.02. In this calculation, the initial magnetic pressure is higher, $\delta_s = \delta_\varphi = 0.1$. It turns out that the filament-like perturbations grow substantially faster. The compressibility f reaches the value $\sim$20 after several revolution and can be still rather large up to the present age. Therefore, the density and magnetic field perturbations are much larger than in previous runs. Most likely, such perturbations will become non-linear already at the very early evolutionary stage.

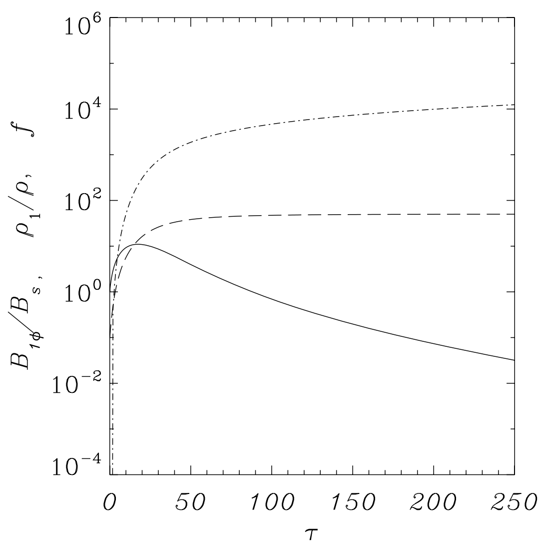

Figure 2. The same as in Fig. 1 but for $\xi = 0.1$.

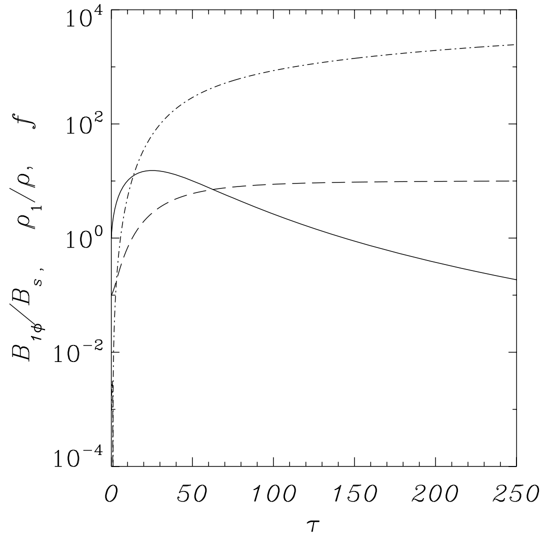

Figure 3. The same dependences as in Fig. 1 but for $\delta_s = \delta_\varphi = 0.1$, $x = 20$, $\mu = 0.0005$, $\xi = 0.01$ and $\varepsilon = 0.1$.

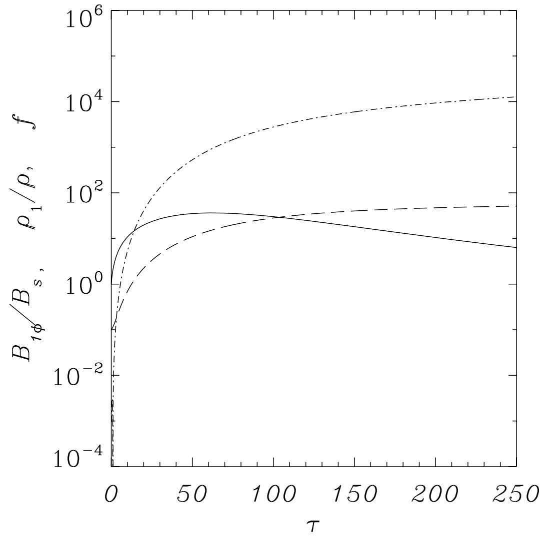

Figure 4. The same dependences as in Fig. 1 but for $\delta_s = \delta_\varphi = 0.0001$, $x = 20$, $\mu = 0.0005$, $\xi = 0.1$ and $\varepsilon = 0.1$.

Fig. 4 plots the evolution of the same filament-like perturbations in the case of a rather weak initial magnetic field, $\delta_s = \delta_\varphi = 0.0001$. The growth is sufficiently fast in this case as well, despite being slower than in Fig. 3. This sort of behaviour is plausible because the last two terms on the left-hand side of equation (21) responsible for the amplification of perturbations depend on the initial magnetic field. However, even in a such weak-baseground field filament-like perturbations become sufficiently large after $\sim$40P to reach a non-linear stage.

In all previous calculations, perturbations were Jeans stable. Consider now the influence of the magnetic field and differential rotation on the behaviour of perturbations which are subject to the Jeans instability as well. We define perturbations as Jeans unstable if the coefficient before $\mathrm{d}^2 f/\mathrm{d}t^2$ in equation (21) is negative at $\tau = 0$,

$$1 + \frac{\delta_s}{\mu} + \delta_\varphi - \frac{\varepsilon}{x^2} < 0, \tag{25}$$

or

$$\lambda > \sqrt{\frac{4\pi}{\rho G}} \left(c_{\rm g}^2 + c_{\rm c}^2 + c_{{\rm A}\varphi 0} + \frac{k^2}{k_z^2} c_{\rm As}^2 \right)^{1/2}, \tag{26}$$

where $\lambda = 2\pi/k$ is the wavelength of perturbations. Taking into account that $c_{\rm g} \sim c_{\rm c} \sim c_{\rm m}$ in galaxies, we can estimate the critical wavelength for the Jeans instability as $\lambda_{\rm cr} = c_{\rm g}\sqrt{4\pi/\rho G}$. If $\lambda > \lambda_{\rm cr}$ then perturbations are Jeans unstable. If we suppose $\rho \sim 10^{-24}$ g cm^{-3} and $c_{\rm g} \sim 8 \times 10^5$ cm s^{-1}, then $\lambda_{\rm cr} \sim 3$ kpc that is larger than the half-thickness of a gaseous disc ($\sim$120 pc). Therefore, the Jeans instability cannot arise in a gaseous disc if $c_{\rm g} = 8$ km s^{-1} and the gas number density is about 1 cm^{-3}. Note, however, that the value 8×10^5 cm s^{-1} corresponds to the averaged gas velocity dispersion of the Milky Way but the sound speed $c_{\rm g}$ can be smaller than the averaged velocity dispersion. Therefore, estimates of $\lambda_{\rm cr}$ can be lower in galaxies. Likely, the Jeans instability can occur in galaxies if the density is higher and the temperature is lower than the averaged values (see e.g. Kuwabara & Ko 2006). We assume that the condition of the Jeans instability (25) is satisfied and consider how the magnetic field and shear influence the behaviour of perturbations with $\lambda > \lambda_{\rm cr}$.

In Fig. 5, we plot the time dependences of perturbations for $\beta_{\rm s} = \beta_\varphi = 0.001$ $\mu = 0.5$, and $\varepsilon = 25.5$. Such perturbations are indeed Jeans unstable because the coefficient before ${\rm d}^2 f/{\rm d}t^2$ in equation (21) is negative at $\tau = 0$ and equal to -0.425. The evolution of perturbations turns out to be qualitatively different in this case: all quantities exhibit oscillations in the contrast to a monotonous growth in the absence of the magnetic field and shear. The period of oscillations is initially comparable to P but becomes shorter with time. The amplitude of density perturbations grows substantially slower because of an oscillatory character of evolution. Nevertheless, after several rotation periods, it becomes of the order of the unperturbed density. To the best of our knowledge, the oscillatory regime of the Jeans instability caused by the magnetic field and shear has never been considered in literature before. Nevertheless, this type of instability can be sufficiently common in galaxies since the magnetic field and shear are among the most important characteristics of the interstellar medium. The oscillatory regime of the Jeans instability could manifest itself in MHD simulations of the galactic evolution as well. In particular, oscillations of the gravitational stress tensor in simulations by Fromang et al. (2004) can be caused, at least, partly by the gravitational instability operating in the oscillatory regime.

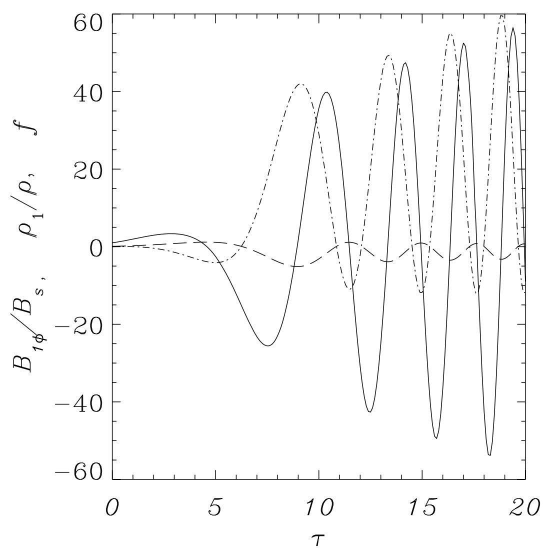

Figure 5. The time behaviour of Jeans unstable perturbations with $\delta_{\rm s} = \delta_\varphi = 0.001, x = 5, \mu = 0.5, \xi = 0.1$ and $\varepsilon = 25.5$.

4 DISCUSSION

We have considered the new local instability of galactic gaseous discs taking account of the magnetic field, differential rotation, and self-gravity. To illustrate the main qualitative features of the instability, we analyzed a particular case of perturbations with the wave-vector $\boldsymbol{k}$ perpendicular to the magnetic field $\boldsymbol{B}$. In this case, the standard magnetorotational instability does not occur because its growth rate is proportional to $(\boldsymbol{k} \cdot \boldsymbol{B})$. The considered instability is related basically to shear and compressibility of a magnetized gas and does not exist in the incompressible limit. This is a principle difference to other well-known instabilities caused by differential rotation such as the Rayleigh or magnetorotational instabilities that can occur in the incompressible limit as well. Note that an attempt to consider instability associated to compressibility of differentially rotating magnetized gas has been undertaken by Blaes & Balbus (1994). These authors, however, analyzed only the unperturbed configuration where the magnetic field has a vertical or azimuthal component, but such configurations are stable in accordance to our consideration.

The necessary condition of this instability is the presence of radial magnetic field and differential rotation. This differs qualitatively from the necessary condition of the magnetorotational instability ($\Omega' < 0$). Therefore, we believe that the considered instability is a new one caused by the combined influence of the magnetic field and shear. The new instability can arise under the conditions when the magnetorotational instability is suppressed, for example, if the magnetic field is sufficiently strong. It is known that the magnetorotational instability does not occur if the magnetic field is stronger than $B_{\rm cr} \sim (\Omega/k)\sqrt{4\pi\rho}$ (e.g. Balbus & Hawley 1992; Urpin & Brandenburg 1998), and this is correct for both axisymmertric and non-axisymmetric perturbations. The instability considered in this paper can occur even if the field is stronger than $B_{\rm cr}$, but it does not arise in the regions where B_φ or $B_{\rm s}$ are vanishing. On the other hand, the magnetorotational instability occurs even in a relatively simple magnetic configurations with $B_{\rm s} = 0$ where the considered instability is suppressed.

Typically, the instability is non-oscillatory if perturbations are Jeans stable and oscillatory in the opposite case. For Jeans unstable perturbations with $\lambda > \lambda_{\rm cr}$, the interplay between the Jeans instability and magnetorotational effects modify qualitatively the classical Jeans instability, and this instability becomes oscillatory. Due to the presence of a radial magnetic field and differential rotation, the Jeans instability can be substantially suppressed. Likely, the instability associated to non-oscillatory modes should be more efficient in galaxies because its growth rate is higher. Besides, non-oscillatory instabilities reach saturation usually at a higher level than the oscillatory ones. This occurs because some fraction of turbulent energy is carried out by propagating waves in the case of oscillatory instabilities.

The criterion for the instability considered can widely be satisfied in galaxies. All galactic discs have both radial and azimuthal components from a spiral or bar perturbation. That is what is observed, and that is what one gets from running numerical simulations. Numerical simulations show that adding spiral perturbations to the galactic model will rapidly generate a radial field even if the magnetic field is purely toroidal in the initial state (Gomez & Cox 2002a,b). The time-scale of such generation is $\sim$100 Myr.

The growth rate of instability depends on the magnetic field strength, differential rotation and the type of perturbations. Generally, the growth rate is sufficient in order the seed perturbations could reach a non-linear regime. The growth of perturbations is not exponential because the background state depends on time. As a result, the growth rate depends on time as well. At the beginning of evolution, the growth rate is comparable to Ω, and this can be faster than the growth rate of combined thermal and magnetorotational instabilities considered by Piontek & Ostriker (2004). The growth time of the considered instability can be even shorter than the growth time of a spiral structure and, therefore, a formation of spiral arms in galaxies is likely accompanied by the generation of turbulent motions and various structures. Relatively large-scale structures caused by the considered instability can contribute, for example, to formation of flocculent spiral structures in galaxies. Elmegreen, Elmegreen & Leitner (2003) suggested that such structures are generated by sheared gravitational instabilities (see e.g. Thomasson, Donner & Elmegreen 1991; Vollmer & Beckert 2002). However, the growth rate of the considered magnetic shear-driven instability can be larger than the growth rate of spiral-forming instabilities that depends sensitively on characteristics of the ISM.

Most likely, the considered instability could not manifest itself in numerical simulations of galactic instabilities done by Kim et al. (2002) and Kim et al. (2003). The authors assume a very simplified magnetic geometry of the unperturbed state (either pure toroidal or vertical magnetic fields that are more suitable for simulations of accretion discs). In both the cases, the instability considered in this paper does not occur. The global 3D MHD simulations of the galactic structure performed by Gomez & Cox (2002a, 2004) also should not indicate the considered instability during the early stage because the authors assume that only the azimuthal field component is non-vanishing in the initial state. Despite the considered instability cannot arise from the very beginning, it can appear during the late stage when a notable radial field is generated. It cannot be excluded, for example, that some loss of regularity in the structure above the mid-plane reported by Gomez & Cox (2004) is due to the instability presented in this paper.

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