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Proton stopping in plasmas considering e^{-}–e^{-} collisions

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Abstract

The purpose of the present paper is to describe the effects of electron-electron collisions on proton electronic stopping in plasmas of any degeneracy. Plasma targets are considered fully ionized so electronic stopping is only due to the free electrons. The stopping due to free electrons is obtained from an exact quantum mechanical evaluation in the random phase approximation, which takes into account the degeneracy of the target plasma. The result is compared with common classical and degenerate approximations. Differences are around 30% in some cases which can produce bigger mistakes in further energy deposition and projectile range studies. We focus our analysis on plasmas in the limit of weakly coupled plasmas then electron-electron collisions have to be considered. Differences with the same results without taking into account collisions are more than 50%.

Keywords: Beam Plasma interaction; Electron collisions; Energy loss; Fast ignition; Plasma degeneracy; Plasma stopping

1. INTRODUCTION

The energy loss of charged particles in a free electron gas is a topic of relevance to understand the details of beam-target interaction processes (Deutsch *et al.*, 1989; Hoffmann *et al.*, 1990, 1994, 2005; Jacoby *et al.*, 1995), especially in the contexts of particle driven fusion, and fast ignition (Deutsch, 1984, 1990, 1992; Deutsch *et al.*, 1989; Eliezer *et al.*, 1995; Roth *et al.*, 2001; Deutsch, 2004; Nardi *et al.*, 2006; Neff *et al.*, 2006). The energy losses of ions moving in an electron gas can be studied with dielectric formalism and random phase approximation (RPA). This approximation consists of considering the effect of the particle as a perturbation, so a linear description of the properties of the target medium can be applied.

RPA is usually valid in the weak coupling limit of an electron gas, i.e., $\Gamma \ll 1$. The coupling parameter, $\Gamma = E_F/(\pi k_F(E_F + k_B T))$ (Arnold & Meyer-ter-Vehn, 1987; Meyer-ter-Vehn *et al.*, 1990), measures the ratio between potential and kinetic energies of the electrons at any degeneracy of the plasma, where E_F and k_F are Fermi energy and Fermi wave number, respectively, and T is the plasma temperature. In this work, we will study plasmas with $\Gamma \leq 1$ so that RPA is not sufficient and the electron collisions of the target gas have to be taken into account. RPA predicts an infinite life-time for target plasma electron collisions, whereas it is well-known that in real materials these excitations are damped. Mermin (1970) derived an expression for the dielectric function taking account of the finite life-time of the collisions.

Mermin (1970) dielectric function has been successfully applied to solids, dense degenerate electron gas (Barriga-Carrasco & Garcia-Molina, 2004), classical plasmas, and nondegenerate electron gas (Barriga-Carrasco, 2006*a*; 2006*b*; Barriga-Carrasco & Maynard, 2006). In this paper, we extend our calculations to consider the effects of electron–electron collisions in RPA for an electron gas of any degeneracy.

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2. DIELECTRIC FORMALISM

Dielectric formalism is based on the dielectric response function of the target material. The dielectric function $\varepsilon(k,\omega)$ is developed in terms of the wave number k and of the frequency ω provided by a consistent quantum mechanical analysis. The dielectric response of the electronic medium is calculated in the RPA. We use atomic units (au), $e = \hbar = m_e = 1$, to simplify formulas.

Dielectric function can be separated into its real and imaginary parts

$$\varepsilon(k,\omega) = \varepsilon_r(k,\omega) + i\varepsilon_i(k,\omega).$$

The real part is (Arista & Brandt, 1984)

$$\varepsilon_r(k,\omega) = 1 + \frac{1}{4z^3\pi k_F}[g(u+z) - g(u-z)], \qquad (1)$$

where $g(x)$ corresponds to

$$g(x) = \int_0^\infty \frac{ydy}{\exp(Dy^2 - \beta\mu) + 1} \ln\left|\frac{1+x}{1-x}\right|,$$

and $u = \omega/kv_F$ and $z = k/2k_F$ are the common dimensionless variables (Lindhard, 1954). $v_F = k_F = \sqrt{2E_F}$ is the Fermi velocity in au, $D = E_F\beta$ is the degeneracy parameter, μ is the chemical potential, and $\beta = 1/k_B T$.

In the limit of high degeneracy, $D \gg 1$,

$$g(x) \approx x + \frac{1}{2}(1 - x^2)\ln\left|\frac{1+x}{1-x}\right|,$$

which substituted in Eq. (1) gives Lindhard dielectric function for a degenerate plasma (Lindhard, 1954). In the opposite limit of high temperatures, $D \ll 1$,

$$g(x) \approx \tfrac{2}{3}D^{1/2}\Theta(D^{1/2}x),$$

where $\Theta(x)$ is the plasma dispersion function (Fried & Conte, 1961)

$$\Theta(x) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} \frac{\exp(-p^2)}{x-p}\, dp,$$

recovering the results for classical plasmas (Peter & Meyer-Ter-Vehn, 1991).

On the other hand, the imaginary part of the dielectric function is

$$\varepsilon_i(k,\omega) = \frac{1}{8z^3 D k_F} \ln\left[\frac{1 + \exp[\beta\mu - D(u-z)^2]}{1 + \exp[\beta\mu - D(u+z)^2]}\right]. \qquad (2)$$

Although this is an exact result for all plasma degeneracy's, the interesting limiting value for high degenerate plasmas for $D \to \infty$

$$\varepsilon_i(k,\omega) \approx \begin{cases} \omega/(8z^3 E_F k_F), & (u \pm z)^2 < 1 \\ [1 - (u-z)^2]/(8z^3 k_F), & (u-z)^2 < 1 < (u+z)^2, \\ 0, & 1 < (u-z)^2 \end{cases}$$

giving rise to the case of degenerate plasma (Lindhard, 1954). For nondegenerate plasmas $D \ll 1$ and $\hbar \to 0$, Eq. (2) transforms into

$$\varepsilon_i(k,\omega) \approx \frac{n_e}{k^3}(2\pi\beta)^{1/2}\exp\left(-\frac{\omega^2\beta}{2k^2}\right),$$

this is the classical result (Peter & Meyer-Ter-Vehn, 1991).

In the dielectric formalism, the electronic stopping for a swift point like ion with charge Z, traveling with constant velocity v through target plasma, defined by its dielectric function, is very well known

$$S_e(v) = \frac{2Z^2}{\pi v^2}\int_0^\infty \frac{dk}{k}\int_0^{kv} d\omega\, \omega\, \mathrm{Im}\left[\frac{-1}{\varepsilon(k,\omega)}\right] \text{(a.u.)}. \qquad (3)$$

For proton velocities $v \gtrsim v_{th}$, where v_{th} is the thermal velocity of the target electrons, the perturbation parameter $\xi = Z/v$ is smaller than one, so the electronic stopping can be determined using RPA.

Figure 1 represents the proton electronic stopping as a function of its velocity in plasmas of different degeneracy, normalized to $S_0 = (Zk_F)^2$. Regarding the plasma degeneracy, the exact stopping is contrasted with both the high degeneracy and the classical limits. We see that by increasing degeneracy parameter of the target, the exact stopping approaches to the high degeneracy limit. On the other hand, decreasing the degeneracy parameter of the target, the exact result approaches the classical limit. As an example, we analyze the electronic stopping for plasma with the same temperature, $T_e = 10$ eV, of Figures 1c and 1d. Figure 1c corresponds to an electron density of $n_e = 10^{22}$ cm^{-3} and Figure 1d to $n_e = 10^{24}$ cm^{-3}. As target electron density increases, degeneracy parameter also increases and it approximates the high degeneracy limit.

3. ELECTRON-ELECTRON COLLISIONS

Mermin (1970) derived an expression for the dielectric function taking account of the finite relaxation time of the electrons

$$\varepsilon_M(k,\omega) = 1 + \frac{(\omega + i\nu)[\varepsilon(k,\omega + i\nu) - 1]}{\omega + i\nu[\varepsilon(k,\omega + i\nu) - 1]/[\varepsilon(k,0) - 1]}, \qquad (4)$$

where ν represents the electron collision frequency. In general, it is contributed from electron–electron and electron–ion collisions. In this paper, we take into account only the electron–electron collisions, in order to avoid considering a dependence of the stopping with the charge and mass of the target ions. It is easy to see that when $\nu \to 0$, the Mermin function reproduces the RPA one. Now it is necessary to calculate the value of this frequency.

The effective collision frequency ν of non-relativistic electrons ($x \ll 1$) was analyzed by Lampe (1968*a*, 1968*b*), using the formalism of the dynamic screening of the electron–electron interaction. $x = v_F/c$ is the relativistic parameter of degenerate electrons and T_F is the Fermi temperature. The expression of ν for the relativistic degenerate elec-

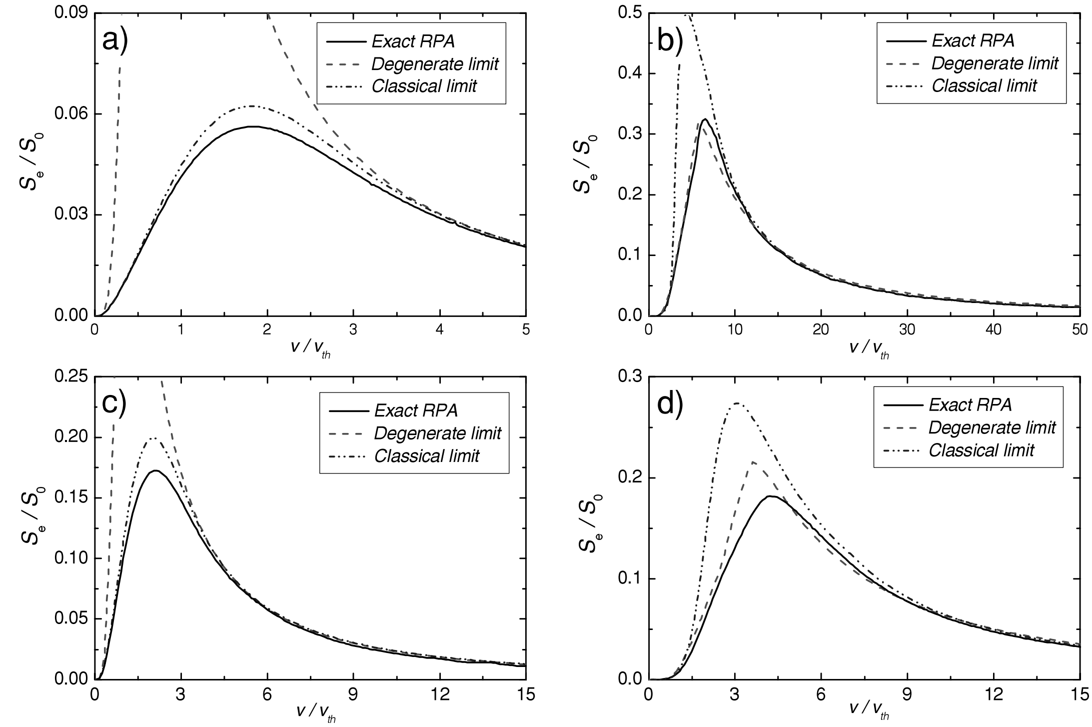

Fig. 1. RPA proton electronic stopping as a function of its velocity in plasmas of different degeneracy. (**a**) $T_e = 100$ eV and $n_e = 10^{23}$ cm^{-3} ($D = 0.079$), (**b**) $T_e = 1$ eV and $n_e = 10^{23}$ cm^{-3} ($D = 7.854$), (**c**) $T_e = 10$ eV and $n_e = 10^{22}$ cm^{-3} ($D = 0.169$), and (**d**) $T_e = 10$ eV and $n_e = 10^{24}$ cm^{-3} ($D = 3.645$). It is compared to degenerate and classical limits. Stopping forces are normalized to $S_\Theta = (Zk_F)^2$.

trons at $T \ll T_p$ was obtained by Flowers and Itoh (1976). Here, T_p is the electron plasma temperature determined by the electron plasma frequency ω_p, $T_p = \hbar\omega_p/k_B$; $\omega_p = \sqrt{4\pi e^2 n_e/(m_e(1+x^2)^{1/2})}$. Urpin and Yakovlev (1980) extended the results of Flowers and Itoh (1976) to higher temperatures, $T < T_F$. In the approximation of static electron screening of the Coulomb interaction, Urpin and Yakovlev (1980) obtained

$$\nu = \frac{3(k_B T)^2}{2\hbar m_e c^2}\sqrt{\frac{\alpha x^3}{\pi^3(1+x^2)^{5/2}}}J(y), \tag{5}$$

where $y = \sqrt{3}T_p/\mathrm{T}$, and α is the fine-structure constant.

Now it is sufficient to calculate the function $J(y)$, presented by Urpin and Yakovlev (1980) as a two-dimensional (2D) integral which depends parametrically on the relativistic parameter x. Lampe (1968*a*, 1968*b*) analyzed this function in the static screening approximation at $x \ll 1$. The asymptotes of J were obtained by Lampe (1968*a*, 1968*b*) for $x \ll 1$ at $y \ll 1$ and $y \gg 1$, by Flowers and Itoh (1976) for $y \gg 1$ at any x, and by Urpin and Yakovlev (1980) for $y \ll 1$ and $x \gg 1$. Timmes (1992) performed calculations and presented a fitting formula for $J(y)$, but it was valid only at $x \gg 1$ and $y < 10^3$. The unified expression of $J(y)$ at $T < T_F$ is valid equally for relativistic and non-relativistic electrons as given by Potekhin *et al.* (1997). They calculated J numerically for a dense grid of x and y in the intervals $0.01 \le x \le 100$ and $0.1 \le y \le 100$. The results are fitted by the expression (Potekhin *et al.*, 1999)

$$J(y) = \left(1 + \frac{6}{5x^2} + \frac{2}{5x^4}\right)\left[\frac{y^3}{3(1+0.07414y)^3} \times \ln\left(\frac{2.810}{y} - \frac{0.810x^2}{y(1+x^2)} + 1\right) + \frac{\pi^5}{6}\frac{y^4}{6(13.91+y)^4}\right], \tag{6}$$

which reproduces also all the asymptotic limits mentioned above. The mean error of the fits is 3.7%, and the maximum error of 11% takes place at $x = 1$ and $y = 0.1$.

Eq. (5) and Eq. (6) are derived for degenerate electrons ($T < T_F$). In the nondegenerate limit, the effective electron–electron collision frequency (the inverse relaxation time), according to the theory of Spitzer–Braginskii (Braginskii, 1957; Spitzer, 1961) and, equals

$$\nu(T \gg T_F) = \frac{8\sqrt{\pi}e^4 n_e \Lambda}{3\sqrt{m_e}(k_B T)^{3/2}}, \tag{7}$$

where $\Lambda\ \alpha\ \ln(T/T_F)$ is the Coulomb logarithm. On the other hand, the collision frequency given by Eq. (5) and Eq. (6),

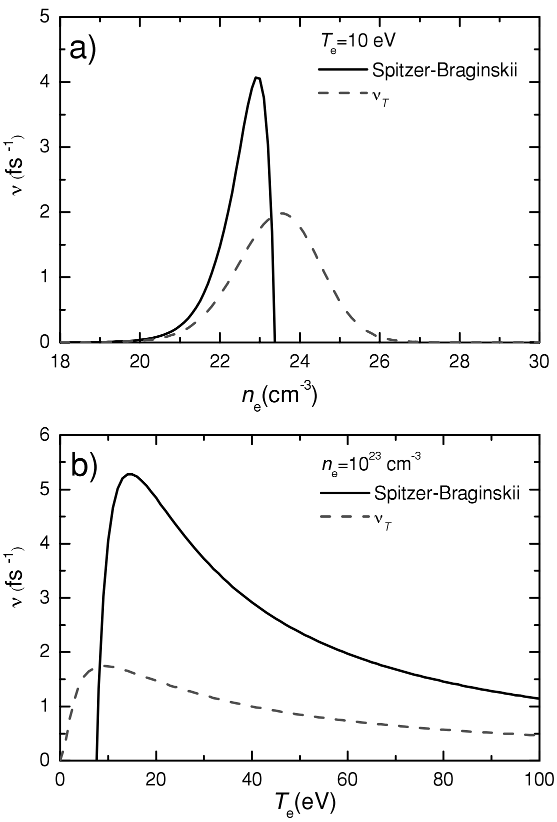

Fig. 2. Collision frequency for a nondegenerate gas, ν_T, as a function of: (**a**) the electron density of the gas with $T_e = 10$ eV and (**b**) the temperature of the gas with $n_e = 10^{23}$ cm^{-3}.

decreases as $T^{-1}\ln T$ instead of the required $T^{-3/2}\ln T$ at $T \gg T_F$.

It turns out, however, that Eqs. (5) and (6) allow a simple generalization for the case of arbitrary degeneracy

$$\nu_T = \frac{\nu}{\sqrt{1 + 0.2T/T_F}}, \quad (8)$$

where ν is the collision frequency of degenerate electrons given by Eqs. (5) and (6). The coefficient 0.2 in this formula ensures a good agreement with the numerical calculations by Hubbard and Lampe (1969), at $T \gg T_F$ (note that the latter calculations are less reliable at $T < T_F$, which shows a comparison with the results of Potekhin *et al.*, 1999).

Figure 2a represents the collision frequency for a nondegenerate gas, ν_T, as a function of the electron density of the gas with a temperature of $T_e = 10$ eV. Figure 2b represents ν_T as a function of the temperature of the gas with an electron density of $n_e = 10^{23}$ cm^{-3}. The effective energy-averaged collision frequency ν_T decreases with T. In fact, it is proportional to $\ln(T)/T^{3/2}$ in the high-T limit, just as in the Spitzer–Braginskii theory.

Now we are going to study the error due to including or not including target e–e$^-$ collisions in the electronic stopping calculation. We introduce the collision frequency ν_T as calculated in Eq. (6) through Mermin dielectric function, Eq. (4). Figure 3 shows proton stopping as a function of its velocity in the same plasma targets as in Figure 1, each one

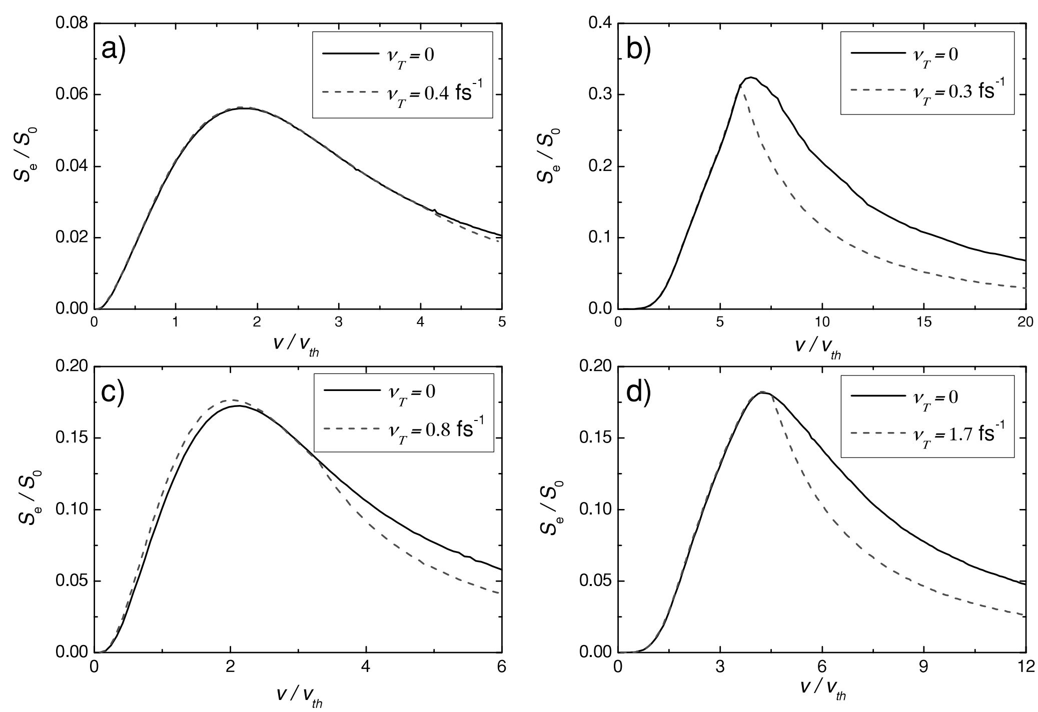

Fig. 3. Proton electronic stopping as a function of its velocity in different plasma targets considering and not considering e$^-$-e$^-$ collisions. (**a**) $T_e = 100$ eV and $n_e = 10^{23}$ cm^{-3}, (**b**) $T_e = 1$ eV and $n_e = 10^{23}$ cm^{-3}, (**c**) $T_e = 10$ eV and $n_e = 10^{22}$ cm^{-3}, and (**d**) $T = 10$ eV and $n_e = 10^{24}$ cm^{-3}.

with different ν_T values. These graphs are obtained by using the quantum mechanical dielectric function with the exact degeneracy. Solid lines correspond to the frequency value $\nu_T = 0$, that is to say, not considering collisions. Dashed lines are the result when we include the collision frequency obtained from Eq. (6) in Eq. (3).

It seems that the first effect of including collisions is increasing the stopping maximum and narrowing the graph at the same time. Also we see that this maximum occurs at smaller or similar velocities than for the calculations without damping. For velocities greater than the velocity at the maximum, the stopping diminishes quite a lot. These facts are more remarkable as the target plasma is more degenerate and coupled (a < c < d < b). Moreover, comparing Figures 3a and 3b, it is seen that the degeneracy or the coupling of the plasma are even more important than the value of the damping frequency. For the most degenerate and coupled plasma, Figure 3b, differences between taking account and not taking account of collisions are around 50%.

4. CONCLUSIONS

The main conclusion of this work is that proton electronic stopping in plasmas in the limit of weakly coupled (T_e = 1–100 eV and $n_e = 10^{22}$–10^{24} cm^{-3}) can not be calculated realistically without using the exact quantum mechanical analysis, which considers the degeneracy of the plasma, and without contemplating electron-electron collisions.

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