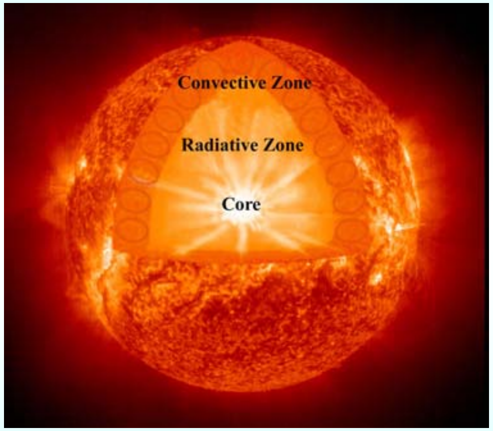
Ioffe Physical-Technical Institute
Department of Theoretical Astrophysics
November, 2009

Plasma Polarization in Massive Astrophysical Objects



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astro-ph:0901.2547

astro-ph:0902.2386

Theses

“Теория строения белых карликов сравнительно проста, хорошо разработана и согласуется с наблюдениями…”

N*

“…Что касается **электростатического потенциала***, **то уделять ему особое внимание не представляется необходимым, потому что*** **трудно себе вообразить какие-либо особенные его проявления***. …”* ***NN****

“… Возможно это правда, что проблема среднего электрического поля получила недостаточно внимания со стороны астрофизиков, но кажется, что это как раз и обусловлено отсутствием его какой-либо роли…

…… Было бы очень интересно … увидеть какие-либо наблюдательные последствия среднего электрического поля …”

NN*

Об электризации, вызванной тяготением массивного тела

“...Из-за малости параметра $\alpha = Gm_p{}^2/e^2$ перечисленные величины исключительно малы, и ***рассматриваемый эффект не может иметь прямых наблюдательных последствий****…”*

NNN*

Basic Idea

Gravitation attracts (heavy) ions and does not attracts electrons.
It leads to a small violation of electroneutrality and polarizes plasma in MAO
(*Sutherland, 1903*)

Polarization field compensates (totally or partially) gravitational (and any other mass-acting) force in thermodynamically equilibrium state
(*macroscopic screening*)

Comment: Ions in thermodynamic equilibrium are suspended, figuratively speaking, in electrostatic field of strongly degenerated and weakly compressed electrons

Expected consequences

Polarization **always** accompanies gravitation

Polarization field must be of the **same order** as gravitation field (*per one proton*)

Polarization field must be **congruent** to gravitation field

Any mass-acting force must be accompanied by polarization

Rotation ⇔ centrifugal force F_c ⇔ ($F_E \sim -\alpha F_c$)

Expansion *or* compression ⇔ inertial force F_a ⇔ ($F_E \sim -\alpha F_a$)

Vibration ⇔ no pure acoustic oscillations ⇔ *(+ electromagnetic oscillations)*

Basic Idea

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Basic statement

(*J. Phys. A: Math. & Theor. 2009*)

New "**Coulomb non-ideality force**" is third "**participant**" in competition between gravitation and polarization forces in equilibrium MAO.

In most cases this new "force" **increases** final **electrostatic field** in comparison with that of ideal-gas solution.

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arXiv:0902.2386v1

Micro- & Macro- Screening

Peter Debye

Erich Hückel

Microscopic screening (*ideal plasma*)

Debye - Hückel screening $(n\lambda^3 << 1)$

Thomas - Fermi screening $(n\lambda^3 >> 1)$

$$F_{av}(\mathbf{r}) = F_{ext}(\mathbf{r}) + F_{scr}(\mathbf{r}) \approx F_{ext}(\mathbf{r})\exp\{-r/r_{scr}\} \to 0$$

$$r \to \infty$$

Macroscopic screening (*ideal plasma*)

Pannekoek - Rosseland screening $(n\lambda^3 << 1)$

Bildsten *et al* screening $(n\lambda^3 >> 1)$

$$F_{av}^{(Z)}(\mathbf{r}) = F_{grav}^{(Z)}(\mathbf{r}) - F_{scr}^{(Z)}(\mathbf{r}) \approx 0$$

What is the problem ?

Micro-scopic screening: **-** *Correct screening for* ***non-ideal*** *plasma at* ***micro-*** *level*

Macro-scopic screening: **-** *Correct screening for* ***non-ideal*** *plasma at* ***macro-*** *level*

$$e\nabla\varphi_{\mathrm{E}}(\mathbf{r}) = -\nabla\varphi_{\mathrm{G}}(\mathbf{r})\frac{\langle \mathrm{Z}|\mathbf{D}_{\mu}^{n}|\mathrm{M}\rangle}{\langle \mathrm{Z}|\mathbf{D}_{\mu}^{n}|\mathrm{Z}\rangle}$$

$$\mathbf{D}_{\mu}^{n} = \left(\mathbf{D}_{\mu}^{n}\right)^{id} + \Delta\mathbf{D}_{\mu}^{n}$$

$\mathbf{D}_{\mu}^{n}$ **- Jacobi matrix** $\left[\!\left[\partial n_j/\partial\mu_k\right]\!\right]_{T,\mu_i(i\neq k)}$ $(j,k = 1,2,3,\ldots)$

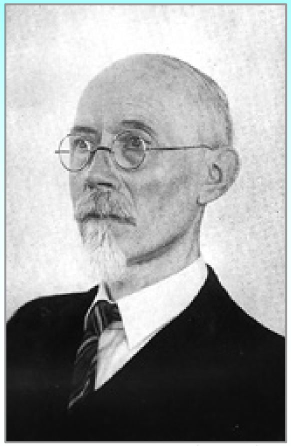
Historical comments

Plasma polarization at micro-level – Debye and Hückel, *Phys. Zeitschr.*, **24**, 8, 1923.

Plasma polarization at macro-level – Pannekoek A., *Bull. Astron. Inst. Neth.*, 1 (**1922**)

= = «» = = – Rosseland S. *Mon. Roy. Astron. Soc.*, **84**, (**1924**)

Pannekoek - Rosseland electrostatic field



A. Pannekoek

Application to plasma:

1) - ideal
2) - non-degenerate
3) - equilibrium
4) - isothermal ($T = const$)
5) - electroneutral

$\{ n_+(r) = n_-(r) \}$

$$dP_e/dr = -GMm_e n_e/r^2 - n_e eE$$

$$dP_i/dr = -GMm_i n_i/r^2 + n_i qE$$

M – mass of the Sun,
G – gravitational constant,
m_e, m_p – electron & proton masses

$$F_E^{(p)} = -(1/2)F_G^{(p)}$$

$$F_E^{(e)} = +(1/2)F_G^{(p)}$$

Generalization to ideal plasma of ions (*A,Z*) and electrons

$$F_E^{(p)} = -\frac{A}{(Z+1)}F_G^{(p)}$$

$$F_E^{(Z)} = -\frac{Z}{(Z+1)}F_G^{(Z)}$$

(*) $F_E^{(p)}$, $F_G^{(p)}$, $F_E^{(Z)}$, $F_G^{(Z)}$, - electrostatic and gravitational forces acting on one proton (p) and ion (A,Z)

Extension to the strongly degenerated plasma

The model of **L. Bildsten** *et al.* (2001 – 2007)

L. Bildsten & D. Hall //*Ap.J.*, 549: (2001) *Gravitational settling of* ^{22}*Ne in liquid white dwarf interior*
P. Chang & L. Bildsten // *Ap.J.*, 585 (2003) *Diffusive nuclear burning in neutron star envelopes*

$$\frac{dP_e}{dr} = -n_e(r)\{m_e g(r) + eE\}$$

$$\frac{dP_i}{dr} = -n_i(r)\{A_i m_p g(r) - Z_i eE\}$$

1) - **ideal**
2) - **strongly degenerated**
3) - isothermal $(T = const)$
4) - electroneutral
$\{ n_+(r) = n_-(r) \}$

5) - equilibrium

The SUN

($p^+ + e^-$)

$$F_E^{(p)} \approx -(1/2)F_G^{(p)}$$

White Dwarf

($_{16}O^{8+}$, $_{12}C^{6+}$, $_{4}He^{2+}$)

$$F_E^{(p)} \approx -2F_G^{(p)}$$

$$\boxed{\boldsymbol{F}_E^{(Z)} \approx -\boldsymbol{F}_G^{(Z)}}$$

With accuracy ~ small parameter x_c

$$x_c \equiv \left(\frac{\partial n_e}{\partial p_e}\right)_T \Big/ \left(\frac{\partial n_i}{\partial p_i}\right)_T$$

NB !

- Average electrostatic field must be of the *same order* as gravitational one*

(* - counting per one proton)

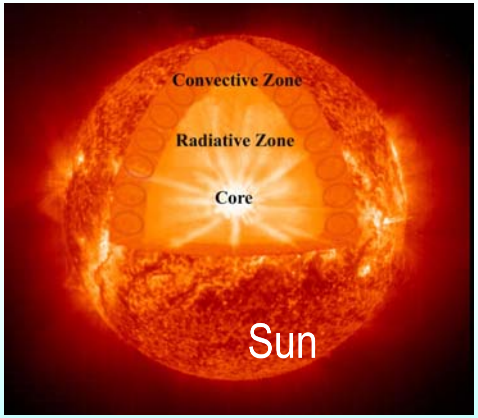
Question: (*Bally & Harrison, 1978*)

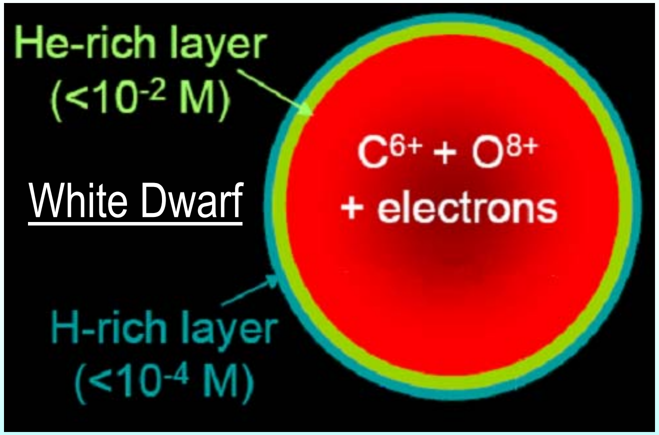
? - **Do both limiting cases** (*ideal non-degenerate and degenerate electrons*) **restrict interval** of **possible *ratio*** of ***gravitational*** and ***electrostatic*** **forces** - **?**

$$F_E^{(p)} = -(1/2)F_G^{(p)}$$

?

$$F_E^{(p)} = -2F_G^{(p)}$$





Answer:

! Yes : - if one takes into account the **electron degeneracy** only **!**

! No : - if one takes into account **non-ideality** effects additionally **!**

(*see below*)

(“*Overcompensation*”)

It may be $\boxed{|F_E^{(p)} / F_G^{(p)}| \geq 2}$ i.e. $\boxed{|F_E^{(Z)} / F_G^{(Z)}| \geq 1}$

Iosilevskiy I. “***Physics of NS***”, S-Pb. Russia, 2008

J. Phys. A, **42**, 2009 // astro-ph:0901.2547

Macroscopic screening *in MAO*

J.Bally & E.Harrison, *AJ, 220, 1978*

The Electrically Polarized Universe

THE ASTROPHYSICAL JOURNAL, **220**: 743–744, 1978 March

THE ELECTRICALLY POLARIZED UNIVERSE

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Received 1977 September 8; accepted 1977 September 22

ABSTRACT

It is shown that all gravitationally bound systems—stars, galaxies, and clusters of galaxies—are positively charged and have a charge-to-mass ratio of ~ 100 coulombs per solar mass. The freely expanding intergalactic medium has a compensating negative charge. The immediate physical consequences of an electrically polarized universe are found to be extremely small.

Subject headings: cosmology — galaxies: intergalactic medium — hydromagnetics

Eddington (1926; see also Rossland 1924) showed in *The Internal Constitution of the Stars* that a star has an internal electric field

$$-\nabla\phi = \alpha(m_p/e)\nabla\psi\,, \tag{1}$$

where ϕ is the electrical potential, ψ is the gravitational potential, m_p is the mass, and e is the charge of a proton. For a nondegenerate electron gas

$$\alpha = \sum n_i A_i \Big/ \sum n_i(1 + Z_i)\,, \tag{2}$$

where the summations are over ion species of density n_i, atomic weight A_i, and effective charge eZ_i. For a fully ionized gas of arbitrary composition, it follows: $\frac{1}{2} \leq \alpha \leqslant 2$. When radiation pressure and electron degeneracy are included, α has similar limits, and in general $\alpha \sim 1$.

From the divergence of equation (1) it is seen that

$$\sigma/\rho = G\alpha m_p/e\,, \tag{3}$$

where σ is the positive gravitationally induced charge density and ρ is the mass density. For a star of total charge Q and mass M the charge-to-mass ratio is

$$Q/M = G\alpha m_p/e\,, \tag{4}$$

and with $\alpha \sim 1$, is of order 100 coulombs per solar mass. This positive charge exists because electrons, despite their low mass, contribute substantially to the pressure, and an electric field is therefore needed to hold in the electron gas. In effect, some electrons escape (most electrons have velocities exceeding the escape velocity), and the remaining electrons are retained by the positively charged star.

It has previously seemed reasonable to suppose that the positive charge within a star is screened by a negatively charged atmosphere containing the expelled electrons. It can be shown, however, that screening occurs in the atmosphere only when the scale height is less than a Debye length.

By allowing for the difference in charge densities in the hydrostatic equations, we find

$$\nabla^2\sigma = -\lambda_D^{-2}(\sigma - G\alpha m_p/e)\,, \tag{5}$$

in place of equation (3), where

$$\lambda_D = (kT/4\pi n_e e^2)^{1/2} \sim 10(T/n_e)^{1/2}\ \text{cm}\,, \tag{6}$$

is the Debye length and n_e is the electron density in a gas of temperature T. Thus, if L is a scale height, and $\nabla^2 \sim L^{-2}$, then equation (3) is recovered whenever $\lambda_D \ll L$. The charge density σ can only become negative in tenuous outer regions of a stellar atmosphere where $\lambda_D > L$, and this only happens when the star and its atmosphere are surrounded by an almost perfect vacuum.

Hence, the positive charge within a star is not automatically screened by a negatively charged atmosphere. The scale length L always greatly exceeds λ_D in stellar atmospheres and the interstellar medium, and both are therefore positively charged and have approximately the same ratio of charge and mass densities as stars. As a rule of thumb we can say that equation (3) applies to all self-gravitating systems of size greater than a Debye length. This leads to the conclusion that an entire galaxy is positively charged. Even elliptical galaxies have a size that is large compared with the Debye length of their interstellar media.

Our equations neglect—among other things—rotational inertial forces and are therefore not correct for rotationally supported gaseous systems. The charge-to-mass ratio of equation (4) does apply, however, to spiral galaxies in which the interstellar gas accounts for only a small fraction of their total mass.

Possibly most galaxies are mem[…]
tionally bound clusters. Since the […]
galaxies is larger than the Debye le[…]
cluster medium (for all conceivable […]
and temperatures), it follows that a[…]
also a charge-to-mass ratio given by[…]

All gravitationally bound system[…]
and clusters of galaxies—are positi[…]
the freely expanding intergalactic […]
clusters of galaxies contains the exp[…]
is therefore negatively charged. St[…]
Sun have center-to-surface potent[…]
$\sim 10^3$ V, giant galaxies have poten[…]
$\sim 10^3$ V, and rich clusters such as […]
have potential differences of $\sim 10^4$[…]

two examples illustrate how small are the physical consequences of an electrically polarized universe.

Blackett (1947) advanced the hypothesis that all massive rotating bodies have magnetic moments of

$$P = \beta G^{1/2}J/c\,, \tag{7}$$

where J denotes angular momentum, c is the speed of light, and β is a dimensionless constant of order unity. In Blackett's words: "It is suggested tentatively that the balance of evidence is that the above equation represents some new and fundamental property of rotating matter." It is now known that numerous astronomical objects (planets, magnetic variable stars, pulsars, etc.) do not obey equation (7) with $\beta \sim 1$. All gravitationally bound systems, however, having the charge-to-mass ratio of equation (4), obey Blackett's relation with

$$\beta \sim (Gm_p^2/e^2)^{1/2} \sim 10^{-18}\,. \tag{8}$$

The magnetic fields generated are exceedingly weak ($\sim 10^{-16}$ gauss in the Sun, and $\sim 10^{-25}$ gauss in the Galaxy) and are generally of no astrophysical interest. Other more effective mechanisms are available for generating seed magnetic fields (Harrison 1970, 1973).

Two charged stars in orbit about each other emit electromagnetic radiation; and if they have different charge-to-mass ratios denoted by α_1, and α_2, then

$$L_{EM}/L_G \sim (\alpha_1 + \alpha_2)^2\beta^2 \sim 10^{-36}\,, \tag{9}$$

where L_{EM} is the magnetic dipole radiation luminosity and L_G is the gravitational radiation luminosity. In the case of electric dipole radiation

$$L_{EM}/L_G \sim (\alpha_1 - \alpha_2)^2\beta^2(cP/a)^2\,, \tag{10}$$

where P is the orbital period and a is the separating distance of the two stars. It is again apparent that the results derived are of no astrophysical importance.

The picture presented consists of positively charged astronomical systems embedded in an intergalactic sea of negative charge. It provides a theoretical basis for Blackett's hypothesis, although the magnetic fields are much weaker than Blackett anticipated. We find the picture of an electrically polarized universe intriguing, and yet, rather surprisingly, we have so far failed to discover any physically significant effects of immediate consequence.

REFERENCES

Blackett, P. M. S. 1947, *Nature*, **159**, 658.
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Harrison, E. R. 1970, *M.N.R.A.S.*, **147**, 279.
———. 1973, *M.N.R.A.S.*, **165**, 185.
Rossland, S. 1924, *M.N.R.A.S.*, **84**, 308.

JOHN BALLY and E. R. HARRISON: University of Massachusetts, Department of Physics and Astronomy, GR Tower B, Amherst, MA 01002

. . . . We find the picture of an electrically polarized universe intriguing, and yet, rather surprisingly, we have so far failed to discover any physically significant effects of immediate consequence.

Widely used approach *(standard)*

From unique equation of hydrostatic (i.e. mechanical) **equilibrium of electro-neutral matter in gravitational field . . .**

$$\frac{dP_\Sigma}{dr} = -\{n_e(r)m_e + n_i(r)m_i\}g(r) = -\rho(r)g(r)$$

. . . to the set of separate equations of hydrostatic equilibrium for each charged specie *(in terms of partial pressures)*

?

$$\frac{dP_e}{dr} = -n_e\{m_e g(r) + eE\}$$

$$\frac{dP_i}{dr} = -n_i\{A_i m_p g(r) - Z_i eE\}$$

What is non-correct ?

NB !

– ***partial pressures*** and ***separate equations of "hydrostatic" equilibrium*** are ***not well-defined*** quantities in ***non-ideal*** plasmas of compact stars

What should be done instead ?

Quasi-stationary state in non-ideal self-gravitating body

(*the problem in general*)

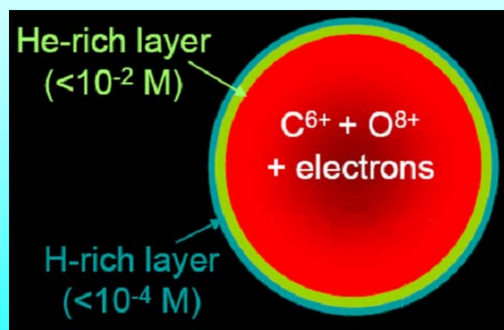
Joint self-consistent description *of* ***thermodynamics*** *and* ***kinetics*** *for* **heat, mass** *and* **impulse transfer** (*diffusion, thermo-conductivity and equation of state*)

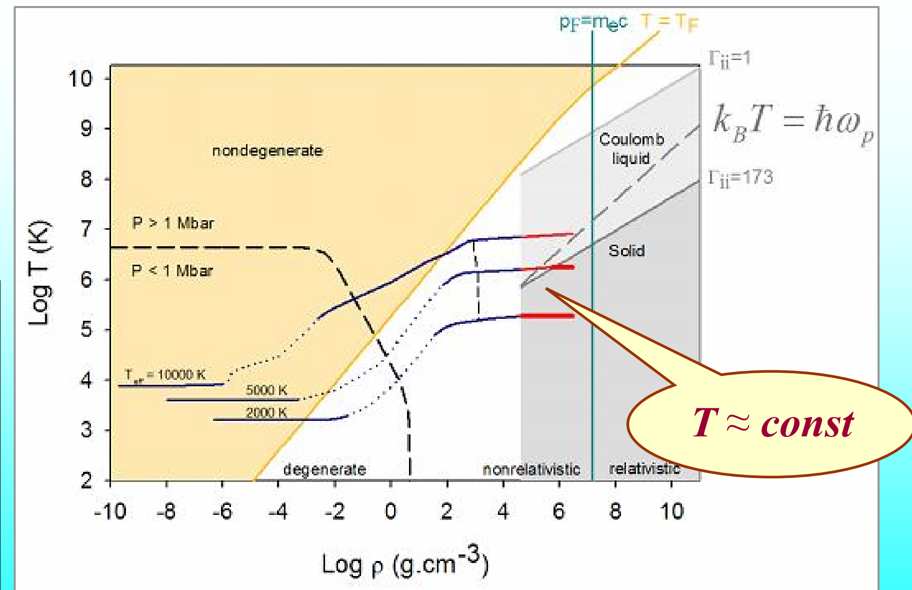
Simplified case

- **Total thermodynamic equilibrium** ($T=$ const)

- **No influence of magnetic field**
- **No relativistic effects**
- **No radiative transfer**

for example:
White Dwarfs





General approach

Variational formulation of equilibrium statistical mechanics

C. De Dominicis,1962 // Hohenberg & Kohn,1964 // R. Evans,1979 etc..

NB !

– ***three small parameters***

$$x_m \equiv \left(m_e / m_i\right)$$

$$x_c \equiv \left(\frac{\partial n_e}{\partial p_e}\right)_T^{id} \Big/ \left(\frac{\partial n_i}{\partial p_i}\right)_T^{id}$$

$$\alpha \equiv \frac{Gm^2}{e^2} \sim 10^{-36}$$

NB !

– ***two large parameters***

- Range of Coulomb forces
- Range of gravitational forces

Integral form of thermodynamic equilibrium conditions

<u>**Variational formulation**</u> (*multi-component version*)

$$F = \min \mathbf{F}\Big[T, V, \{N\} \,|\, \{n_j(\mathbf{r})\} : \{n_{jk}(\mathbf{r},\mathbf{r}')\} \ldots\Big]^{\{T=const,\, N_k=const\}}_{V_1(\mathbf{r}),V_{1,2}(\mathbf{r},\mathbf{r}'),V_{1,2,3}(\mathbf{r},\mathbf{r}',\mathbf{r}'')\ldots=const}$$

The main problem – ***<u>strong non-locality</u>*** of the **free energy functional** due to **long-range nature** of **Coulomb** and **gravitational interaction**

<u>**Standard**</u>: separation of main non-local parts.

$$F\{T, V(r) / \big[\{n_i(\cdot)\} / \{n_{ij}(\cdot,\cdot)\}\big] \equiv$$

$$\equiv -\sum_{jk} \frac{G m_j m_k}{2} \int \frac{n_j(\mathbf{r}) \cdot n_k(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r}\, d\mathbf{r}' + \sum_{jk} \frac{Z_j Z_k e^2}{2} \int \frac{n_j(\mathbf{r}) \cdot n_k(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r}\, d\mathbf{r}' + F^*\big[\{n_i(\cdot)\} // \{n_{ij}(\cdot,\cdot)\}\big]$$

NB! The rest *F**{*…*} is the free energy of *<u>new</u>* system on *<u>compensating background</u>(s)*

It's assumed that the rest free energy functional $\boldsymbol{F}^*[n_i // n_{ij}]$ is *<u>weakly non-local</u>*

Hence *<u>weakly non-local chemical potentials</u>*: $\mu_j^{(\mathrm{chem})}$ - could be introduced

$$\mu_j^{(chem)} \equiv \Big(\delta F^*[\cdots] / \delta n_j(\cdot)\Big)_{T, n_{k\neq j}}$$

Local forms of thermodynamic equilibrium conditions

<u>Heat exchange</u>:

$$T(\mathbf{r}) = \text{const}$$

<u>Impulse exchange</u>:

$$\nabla P_\Sigma = -\rho(\mathbf{r})\nabla\varphi_G(\mathbf{r})$$

<u>Particle exchange</u>:

<u>In terms of potentials</u>

Constance of total (generalized) ***<u>electro-chemical</u>*** **potential**

$$m_j\varphi_{\mathrm{G}}(\mathbf{r}) + q_j\varphi_{\mathrm{E}}(\mathbf{r}) + \mu_j^{(\mathrm{chem})}\{n_{\mathrm{i}}(\mathbf{r}),\ n_{\mathrm{e}}(\mathbf{r}),\ \{n_{jk}(\mathbf{x},\mathbf{y})\}\ T\} = \text{const}$$

(j,k = electrons, ions)

<u>In terms of forces</u>

Balance of **forces** including **generalized "*<u>non-ideality</u>*" force**

$$m_j\nabla\varphi_{\mathrm{G}}(\mathbf{r}) + q_j\nabla\varphi_{\mathrm{E}}(\mathbf{r}) + \nabla\mu_j^{(\mathrm{chem})}\ \{n_{\mathrm{i}}(\mathbf{r}),\ n_{\mathrm{e}}(\mathbf{r}),\ \{n_{\mathrm{jk}}(\mathbf{x},\mathbf{y})\}\ T\} = 0$$

(j,k = electrons, ions)

$\varphi_{\mathrm{G}}(\mathbf{r})$ и $\varphi_{\mathrm{E}}(\mathbf{r})$ – *gravitational and electrostatic potentials*

<u>NB</u> !

The set of equations for *<u>electro-chemical potentials</u>* instead of the set of separate equations of "hydrostatic" equilibrium for partial pressures !

$$F = \min F\left(T, V, \{N_k\}/\{n_i(\cdot)\}/\{n_{ij}(\cdot,\cdot)\}\right) \equiv$$

$$\equiv - \sum_{jk} \frac{G m_j m_k}{2} \int \frac{n_j(\mathbf{r}) \cdot n_k(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r} d\mathbf{r}' + \sum_{jk} \frac{Z_j Z_k e^2}{2} \int \frac{n_j(\mathbf{r}) \cdot n_k(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r} d\mathbf{r}' + F^*\left[\{n_i(\cdot)\}/\{n_{ij}(\cdot,\cdot)\}\right]$$

NB! *Extremely low* strength of *gravitational* interaction in comparison with *Coulomb* one

$$\alpha \equiv \frac{Gm^2}{e^2} \sim 10^{-36}$$

extremely small parameter !

Even *extremely small* deviation from electroneutrality in *Coulomb term* leads to significant *energy variation* in free energy functional

Extremely small but *non-zero* violation of global electroneutrality !

Total charge disbalance - ΔQ

$\Delta Q \sim \alpha N_{\Sigma}^{barion}$ $\quad$ $N_{\Sigma}^{barion} \approx 10^{57}$ $\quad$ $\Delta Q \sim \alpha \cdot 10^{57} \approx \left(10^{21} - 10^{22}\right) e$ $\quad$ ≈ 100 Q

Equilibrium plasma is electroneutral *almost everywhere*

NB! Deviation from electroneutrality *must not* be uniform *totally everywhere*

Exceptions: - discontinuity surfaces

(*phase boundaries, jump-like change in ionic composition etc.*)

Macroscopic Screening in Non-Ideal Plasma

In electroneutrality regions one obtains:

$$\boxed{e\nabla\varphi_{\mathrm{E}}(\mathbf{r}) = -\nabla\varphi_{\mathrm{G}}(\mathbf{r})\frac{\langle \mathrm{Z}|\mathbf{D}^{n}_{\mu}|\mathrm{M}\rangle}{\langle \mathrm{Z}|\mathbf{D}^{n}_{\mu}|\mathrm{Z}\rangle}}$$

astro-ph:0902.2386

Here:

$\mathbf{D}^{n}_{\mu}(\mathbf{r})$ matrix $\Leftrightarrow$ $\{\delta\mathbf{n}(\mathbf{r})/\delta\boldsymbol{\mu}(\mathbf{r}')\}_T \equiv \left[\!\left[\delta n_j(\mathbf{r})/\delta\mu_k(\mathbf{r}')\right]\!\right]_{T,\mu_i(i\neq k)} \equiv \left[\!\left[\delta^2 F^*/\delta\mu_j(\mathbf{r})\delta\mu_k(\mathbf{r}')\right]\!\right]_{T,\mu_i(i\neq k)}$

$\langle\mathbf{Z}| \equiv \{Z_j\}$

$|\mathbf{M}\rangle \equiv \{M_j\}$

$\mathbf{D}^{n}_{\mu}$ **is inverse matrix to:** $\mathbf{D}^{\mu}_{n} \equiv \left[\!\left[\delta^2 F^*/\delta n_j(\mathbf{r})\delta n_k(\mathbf{r}')\right]\!\right]_{T,n_i(i\neq k)}$

$$\mathbf{D}^{n}_{\mu} * \mathbf{D}^{\mu}_{n} = \mathbf{E}$$

Non-ideality effects $\Leftrightarrow$ $\mathbf{D}^{n}_{\mu} = \left(\mathbf{D}^{n}_{\mu}\right)^{id} + \Delta\mathbf{D}^{n}_{\mu}$

- **Total thermodynamic equilibrium** (*T* = const)

- **No influence of magnetic field**
- **No relativistic effects**
- **No radiative transfer**

$$\boxed{e\nabla\varphi_{\mathrm{E}}(\mathbf{r}) = -\nabla\varphi_{\mathrm{G}}(\mathbf{r})\frac{\langle \mathrm{Z}|\mathbf{D}^{n}_{\mu}|\mathrm{M}\rangle}{\langle \mathrm{Z}|\mathbf{D}^{n}_{\mu}|\mathrm{Z}\rangle}}$$

Does not restricted by:

Spherical symmetry condition
Nomenclature of ions
Degree of ionization
Degree of Coulomb non-ideality
Degree of electronic degeneracy
....

NB! *Matrix* $\mathbf{D}^{n}_{\mu}$ *is still non-local*

$$F = \min F\left(T, V, \{N_k\} / \{n_i(\cdot)\} / \{n_{ij}(\cdot,\cdot)\}\right) \equiv$$

$$\equiv -\sum_{jk} \frac{G m_j m_k}{2} \int \frac{n_j(\mathbf{r}) \cdot n_k(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r} d\mathbf{r}' + \sum_{jk} \frac{Z_j Z_k e^2}{2} \int \frac{n_j(\mathbf{r}) \cdot n_k(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r} d\mathbf{r}' \boxed{+ F^*\left[\{n_i(\cdot)\} / \{n_{ij}(\cdot,\cdot)\}\right]}$$

"Quasi-uniformity" Approximation

$$F = \min F\left(T, V, \{N_k\} / \{n_i(\cdot)\}\right) \equiv$$

$$\equiv -\sum_{jk} \frac{G m_j m_k}{2} \int \frac{n_j(\mathbf{r}) \cdot n_k(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r} d\mathbf{r}' + \sum_{jk} \frac{Z_j Z_k e^2}{2} \int \frac{n_j(\mathbf{r}) \cdot n_k(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r} d\mathbf{r}' \boxed{+ \int f^*\left(\{n_i(\mathbf{r}) \ldots n_k(\mathbf{r})\}\right) d\mathbf{r}}$$

μ *is a function, not functional*

$$\mu_j^{(chem)}(\mathbf{r}) \equiv \left(\partial f^*[T, \{n_k(\mathbf{r})\}] / \partial n_j\right)_{T, n_{k \neq j}}$$

$$\boxed{f^*(\{n\}) \equiv \lim \left\{ \frac{F(N_i \ldots N_k, V, T)}{V} \right\}_{\{N_k\}, V \to \infty}^{N_k / V \to n_k}}$$

In terms of potentials

$$m_j \varphi_{\mathrm{G}}(\mathbf{r}) + q_j \varphi_{\mathrm{E}}(\mathbf{r}) + \mu_j^{(\mathrm{chem})}[\{n_k(\mathbf{r})\}, T] = \mathrm{const} \qquad (j,k = \text{electrons, ions})$$

In terms of forces

$$m_j \nabla \varphi_{\mathrm{G}}(\mathbf{r}) + q_j \nabla \varphi_{\mathrm{E}}(\mathbf{r}) + \nabla \mu_j^{(\mathrm{chem})}[\{n_k(\mathbf{r})\}, T] = 0 \qquad (j,k = \text{electrons, ions})$$

NB! The *local* free energy density $f^*(\{n\})$ must be defined for *non-electroneutral* densities $\{n_k\}$

$$F = \min F\left(T, V, \{N_k\}/\{n_i(\cdot)\}/\{n_{ij}(\cdot,\cdot)\}\right) \equiv$$

$$\equiv \cancel{\sum_{jk} \frac{Z_j Z_k e^2}{2} \int \frac{n_j(\mathbf{r}) \cdot n_k(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r} d\mathbf{r}'} - \sum_{\mathrm{jk}} \frac{G m_j m_k}{2} \int \frac{n_j(\mathbf{r}) \cdot n_k(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r} d\mathbf{r}' + F^*\left[\{n_i(\cdot)\}/\{n_{ij}(\cdot,\cdot)\}\right]$$

Standard*

$$Q(\mathbf{r}) \equiv \sum_j Z_j e n_j(\mathbf{r}) \equiv 0 \longrightarrow \boxed{\nabla \varphi_{\mathrm{E}}(\mathbf{r}) \equiv 0}$$

$$F = \min F\left(T, V, \{N_k\}/\{n_i(\cdot)\}\right) \equiv - \sum_{\mathrm{jk}} \frac{G}{2} \int \frac{\rho_j(\mathbf{r}) \cdot \rho_k(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r} d\mathbf{r}' + F^*\left[\{n_i(\cdot)\}\right]$$

NB ! The free energy $F^*\{\ldots\}$ is still *non-local*

Quasi-uniformity approximation

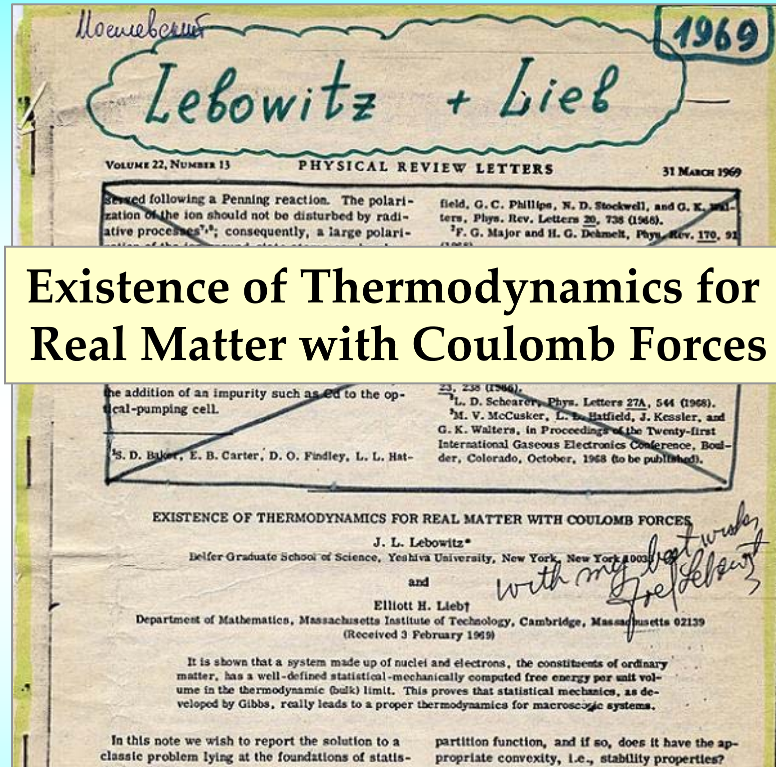
$$F = \min F\left(T, V, \{N_k\}/\{n_i(\cdot)\}\right) \equiv - \sum_{\mathrm{jk}} \frac{G}{2} \int \frac{\rho_j(\mathbf{r}) \cdot \rho_k(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r} d\mathbf{r}' + \int f^*\left(\{n_i(\mathbf{r}) \ldots n_k(\mathbf{r})\}\right) d\mathbf{r}$$

(*) Shapiro S.L., Teukolsky S.A. // Black Holes, White dwarfs and Neutron stars / 1983, p.

The problem of thermodynamic limit in Coulomb system

Lebowitz J.L. & Lieb E.H. *PRL*, **22** 631 (1969)

$$f^*\left(\{n\},T\right) \equiv \lim \left\{ \frac{F\left(N_i \ldots N_k, V, T\right)}{V} \right\}_{\{N_k\},V\to\infty}^{N_k/V \to n_k}$$


Lebowitz + Lieb

1969

VOLUME 22, NUMBER 13 PHYSICAL REVIEW LETTERS 31 MARCH 1969

EXISTENCE OF THERMODYNAMICS FOR REAL MATTER WITH COULOMB FORCES

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(Received 3 February 1969)

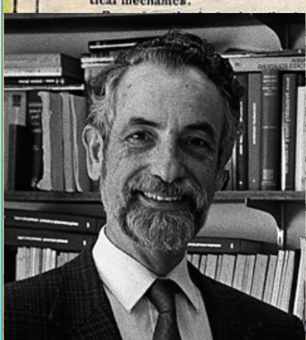
It is shown that a system made up of nuclei and electrons, the constituents of ordinary matter, has a well-defined statistical-mechanically computed free energy per unit volume in the thermodynamic (bulk) limit. This proves that statistical mechanics, as developed by Gibbs, really leads to a proper thermodynamics for macroscopic systems.

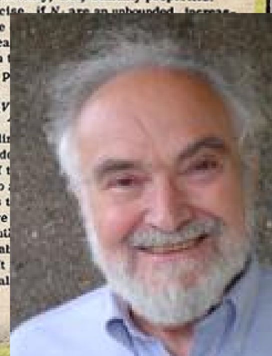
In this note we wish to report the solution to a classic problem lying at the foundations of statistical mechanics.

partition function, and if so, does it have the appropriate convexity, i.e., stability properties?

with my best wishes
Joel Lebowitz

Existence of Thermodynamics for Real Matter with Coulomb Forces


Elliot Lieb


Joel Lebowitz

Disbalance of net electric charge is the first source of conditional nature of thermodynamic limit in Coulomb system

Thermodynamic limit strongly depends on disbalance of net electric charge

$Q \to 0$ | $Q \sim N^{\varepsilon}\ (< 2/3)$ | $Q \sim N^{\varepsilon}\ (> 2/3)$

Surface dipole is the second source of conditional nature of thermodynamic limit in Coulomb system

Let us use the *Electroneutral* *Grand Canonical Ensemble*

“Surface dipole” is the second source of non-locality for thermodynamics in equilibrium Coulomb system

Galvani potential

Any phase boundary in equilibrium Coulomb system is accompanied by existence of
stationary electrostatic potential difference
due to the **long-range nature** of **Coulomb** forces

Iosilevskiy & Chigvintsev, *J. Physique* (2000)

Basic question:

Do both these mechanisms (disbalance *of* net electric charge *and* surface dipole) **exhaust all non-locality** *of* **free energy functional** $F^*[\{n_i\}/\{n_{ik}\}]$ **or not ?**

The main problem still is the ***non-locality*** of **free energy functional** due to **long-range nature** of **Coulomb** and **gravitational interaction**

Applications

Details of Variational Procedure

$$F = \min F\left(T, V, \{N_j\} / \{n_i(\cdot)\} / \{n_{ij}(\cdot,\cdot)\}\right) \equiv$$

$$\equiv - \sum_{jk} \frac{G m_j m_k}{2} \int \frac{n_j(\mathbf{r}) \cdot n_k(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r} d\mathbf{r}' + \sum_{jk} \frac{Z_j Z_k e^2}{2} \int \frac{n_j(\mathbf{r}) \cdot n_k(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r} d\mathbf{r}' + F^*\left[\{n_i(\cdot)\} / \{n_{ij}(\cdot,\cdot)\}\right]$$

Dilemma: ***Physical*** *or* ***Chemical*** representation **?**

Physical *picture* ⇔ ***Chemical*** *picture*

Basic Units → **Nuclei** *and* **electrons**

Atoms, molecules . . .
free ions *and* **free electrons**

Planets, BD, ...

$H^{+} + He^{++} + e^{(-)}$

$H + H_2 + H^{(-)} + H_2^{+} + H^{+} + He + He^{+} + He^{++} + \ldots + e^{(-)}$

NB !

In each point Saha-like equations are valid !

$AB \Leftrightarrow A + B$ → $\mu_{AB}(\mathbf{r}) = \mu_A(\mathbf{r}) + \mu_B(\mathbf{r})$

Saha-like equations for local parameters

Dilemma: *Physical* or *Chemical* representation ?

***Physical** picture* ⇔ ***Chemical** picture*

Nuclear Plasma

Basic Units →

n, *p*, and **electrons**

*n**, *p**, *N(A,Z)* and **electrons***

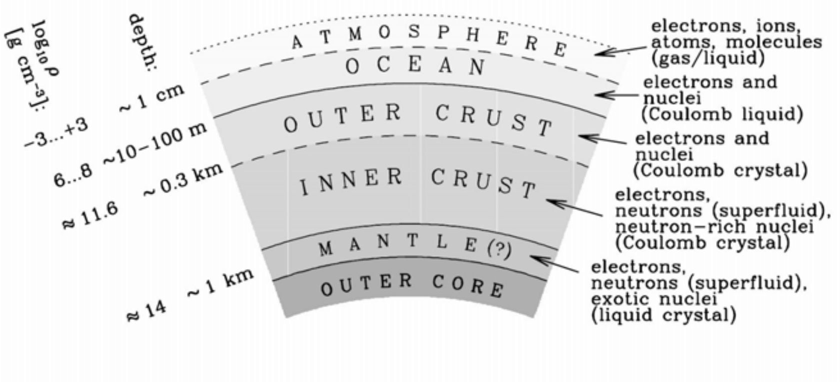


Figure 2.1. Schematic structure of an envelope of a neutron star with the internal temperature $\sim 10^8$ K (see text for more details).

Haensel P., Potekhin A., Yakovlev D.
Neutron Stars, Springer, New York, 2007

"Free" neutrons, protons
*and their "***clusters***"*

Typel S., Roepke G., Klahn T., Blaschke D., and Wolter H. arXiv:0908.2344v1

$$N(A,Z) \;\Leftrightarrow\; Zp + (A - Z)n$$

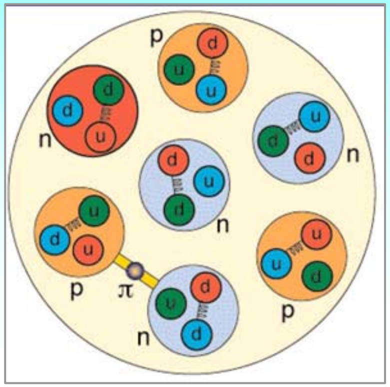
Saha-like equations are valid !

$$\mu_{N(A,Z)}(\mathbf{r}) = Z\mu_{\mathrm{p}}(\mathbf{r}) + (A - Z)\mu_{n}(\mathbf{r})$$

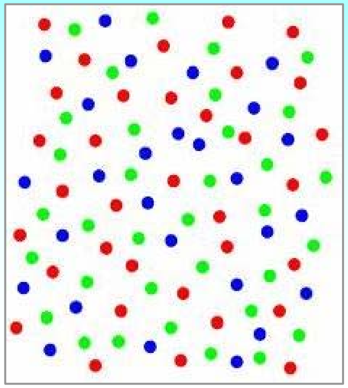
Dilemma: *Physical* or *Chemical* representation ?

Physical picture ?

Strange (hybrid) stars



⇔



u, d, s, p, n, e

$\mu_u, \mu_d, \mu_s, \mu_p, \mu_n, \mu_e$

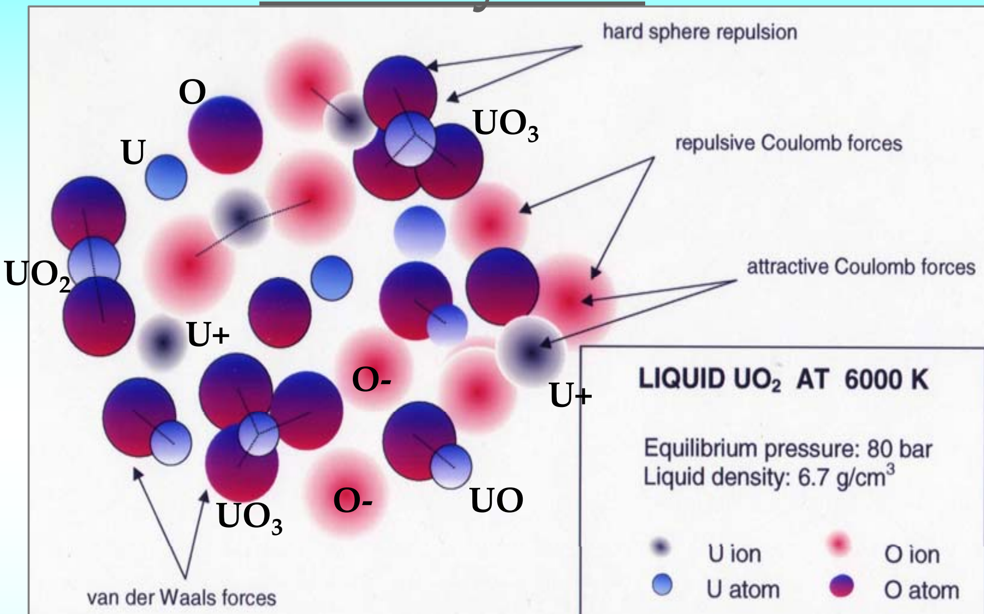
u + e ⇔ d
d ⇔ s
p + e ⇔ n
n ⇔ u + 2d
(p ⇔ 2u + d)

$$\mu_u + \mu_e = \mu_d,$$
$$\mu_d = \mu_s,$$
$$\mu_p + \mu_e = \mu_n \equiv \mu_B,$$
$$\mu_n = \mu_u + 2\mu_d,$$
$$(\mu_p = 2\mu_u + \mu_d)\,.$$

Endo T., Maruyama T., Chiba S., Tatsumi T.
astro-ph/0601017v1/ 2006 /

Chemical picture

U – O system



Multi-molecular model
(*Liquid & Gas*)
$U + O + O_2 + UO + UO_2 + UO_3$
$U^+ + UO^+ + UO_2^+ + O^- + UO_3^- + e^-$

$U + 2O \Leftrightarrow UO_2$
$2O \Leftrightarrow O_2$
$U^+ + e \Leftrightarrow U$
$UO_3 + e \Leftrightarrow UO_3^-$
……

$\mu_U + 2\mu_U = \mu_{UO2}$
$2\mu_O = \mu_{O2}$
$\mu_{U+} + \mu_e = \mu_U$
$\mu_{UO3} + \mu_e = \mu_{UO3-}$
…………

Iosilevskiy I. / "*Physics of Neutron Stars*", S-Pb. Russia, 2008

$$e\nabla\varphi_{\mathrm{E}}(\mathbf{r}) = -\nabla\varphi_{\mathrm{G}}(\mathbf{r})\frac{\langle \mathrm{Z}|\mathbf{D}_{\mu}^{n}|\mathrm{M}\rangle}{\langle \mathrm{Z}|\mathbf{D}_{\mu}^{n}|\mathrm{Z}\rangle}$$

Can be solved in simplified cases:

- Ideal-mixture approximation

(*multi-component "chemical picture*")

- Classical weakly non-ideal plasma

(*Debye approximation in Grand Canonical Ensemble*)

- Strongly non-ideal ionic mixture on strongly degenerated weakly non-ideal electrons

(*switching-off the electron-ionic correlations*)

- Two-component electron-ionic system with arbitrary degree of degeneracy and non-ideality

(*strongly correlated system*)

Ideal-mixture approximation

$$\mathbf{D}_\mu^n = \left(\mathbf{D}_\mu^n\right)^{id}$$

(*chemical picture: - a, b, ab, ab$_2$, a$_2$b, . . . a$_n$b$_m$*)

$$\boxed{e\nabla\varphi_{\mathrm{E}}(\mathbf{r}) = -\nabla\varphi_{\mathrm{G}}(\mathbf{r})\frac{\langle \mathrm{Z}|\mathbf{D}_\mu^n|\mathrm{M}\rangle}{\langle \mathrm{Z}|\mathbf{D}_\mu^n|\mathrm{Z}\rangle}} \Leftrightarrow \boxed{e\nabla\varphi_{\mathrm{E}}(\mathbf{r}) = -\nabla\varphi_{\mathrm{M}}(\mathbf{r})\frac{\left(\sum_j \tilde{n}_j M_j Z_j\right)}{\left(\sum_j \tilde{n}_j Z_j^2\right)}}$$

$$\langle \mathbf{Z}| \equiv \{Z_j\} \qquad |\mathbf{M}\rangle \equiv \{M_j\}$$

$$\tilde{n}_j \equiv kT\left(\partial n_j / \partial\mu_j\right)_{T,n_{k\neq j}}^{id.gas} \quad (j = 1,2,3\ldots)$$

$$\tilde{n}_e \to 0 \qquad \left(n_e\lambda_e^3 \gg 1\right)$$

NB ! Electronic contribution falls out from $\boxed{e\nabla\varphi_{\mathrm{E}}(\mathbf{r}) = -\nabla\varphi_{\mathrm{M}}(\mathbf{r})\frac{\left(\sum_j \tilde{n}_j M_j Z_j\right)}{\left(\sum_j \tilde{n}_j Z_j^2\right)}}$ in the limit of strong electron degeneracy due to diminishing of ideal-gas electronic compressibility:

Here:

$$\mathbf{D}_\mu^n(\mathbf{r}) \Leftrightarrow \{\delta\mathbf{n}(\mathbf{r})/\delta\boldsymbol{\mu}(\mathbf{r}')\}_T \equiv \left[\!\left[\delta^2 F^*/\delta\mu_j(\mathbf{r})\delta\mu_k(\mathbf{r}')\right]\!\right]_{T,\mu_i(i\neq k)} \qquad \mathbf{D}_\mu^n * \mathbf{D}_n^\mu = \mathbf{E}$$

$\mathbf{D}_\mu^n$ is inverse to: $\mathbf{D}_n^\mu \equiv \left[\!\left[\delta^2 F^*/\delta n_j(\mathbf{r})\delta n_k(\mathbf{r}')\right]\!\right]_{T,n_i(i\neq k)}$

Classical weakly non-ideal plasma

(*Debye approximation in Grand Canonical Ensemble*)

Coulomb "non-ideality force" moves positive ions *inside* the star in addition to gravitation

Hence "non-ideality force" *increases* compensating electrostatic field $\varphi_{\mathrm{E}}(r)$

in comparison with the ideal-gas approximation

Classical weakly non-ideal *i-e* plasma

(*Debye approximation*)

$$F_{\mathrm{G}}^{(Z)} \approx -F_{\mathrm{E}}^{(Z)} \left[1 + \frac{(1 - Z^2 \Gamma_{\mathrm{D}}/4)}{Z(1 - \Gamma_{\mathrm{D}}/4)} \right],$$

$$\Gamma_{\mathrm{D}} \equiv (e^2/kT r_D) \ll 1, \quad \left\{ r_{\mathrm{D}}^{-2} \equiv (4\pi e^2 (1 + Z^2)/kT) \right\}. \qquad \zeta_e \equiv n_e \lambda_e^3 \ll 1$$

Non-ideality effects in two-component plasma

$\{+Z, \mathrm{e}\}$

Equilibrium condition with "*non-ideality force*"

$$m_k \nabla \varphi_{\mathrm{G}}(\mathbf{r}) + Z_k e \nabla \varphi_{\mathrm{E}}(\mathbf{r}) + \nabla \mu_k^{(\mathrm{chem})}\{n_{\mathrm{i}}(\mathbf{r}), n_{\mathrm{e}}(\mathbf{r}), T\} = 0 \qquad (k = \text{electrons, ions})$$

Final equation for average electrostatic field

(*with taking into account non-ideality and degeneracy effects*)

$$\boxed{m_i \nabla \varphi_G(\mathbf{r}) + Z_i e \nabla \varphi_E(\mathbf{r}) \left[1 + \frac{(\mu_{ii}^0 + \Delta_i^i + Z\Delta_e^i)}{Z(Z\mu_{ee}^0 + Z\Delta_e^e + \Delta_i^e)} \right] = 0}$$

Here:

$\mu_j^0(n_j, T)$ – ideal-gas part of (*local*) chemical potential of specie j

$\Delta\mu_j^{(chem)}(n_j, n_i \ldots n_k, T)$ – non-ideal-gas part of (*local*) chemical potential of specie j

$$\mu_{jj}^0 \equiv \left(\frac{\partial \mu_j^0}{\partial n_j} \right) \qquad \Delta_k^j \equiv \left(\frac{\partial \Delta\mu_j}{\partial n_k} \right)$$

Non-ideality effects in two-component plasma $\{+Z, \mathrm{e}\}$

(summary)

1) Ideal and **non-degenerate gas** $(n\lambda_e^3 << 1)$

$$F_G^{(Z)} + 2F_E^{(Z)} = 0$$

Polarization compensates just *one half* of gravitational attraction (*for symmetric ion A=2Z*)

2) Non-ideal and **non-degenerate gas** $(n\lambda_e^3 << \mathbf{1})$

Polarization compensates *more* than *one half* of gravitational attraction (*for symmetric ion*)

$$F_G^{(Z)} + F_E^{(Z)}[2 - \varepsilon(\Gamma)] = 0 \qquad 0 < \varepsilon(\Gamma) < 1$$

3) Ideal and **highly-degenerate gas** $(n\lambda_e^3 >> 1)$

$$F_G^{(Z)} + F_E^{(Z)} \simeq 0$$

Polarization compensates gravitational attraction of ions *almost totally*

4) Non-ideal and **highly-degenerate gas** $(n\lambda_e^3 >> \mathbf{1})$

$$F_{\mathrm{E}}^{(Z)} + F_{\mathrm{G}}^{(Z)}[1 + \varepsilon(\Gamma, n\lambda^3)] = 0$$

Polarization compensates *not only* gravitational attraction
but additional "*non-ideality force*" directed towards the center of a star **!**

«Global» non-ideality effect !

Quickly rotating star

(*addition of centrifugal force*)

Constance of total (generalized) *electro-chemical* potential

$$m_j\,\{\varphi_G(\mathbf{r}) + \varphi_C(\mathbf{r})\} + q_j\,\varphi_E(\mathbf{r}) + \mu_j^{(\mathrm{chem})}\{n_i(\mathbf{r}),\, n_e(\mathbf{r}),\, \{n_{jk}(\mathbf{x},\mathbf{y})\}\; T\} = \mathrm{const}$$

(j,k = electrons, ions)

Balance of **forces** including **generalized "*non-ideality*" force**

$$m_j\{\nabla\varphi_G(\mathbf{r}) + \nabla\varphi_C(\mathbf{r})\} + q_j\nabla\varphi_E(\mathbf{r}) + \nabla\mu_j^{(\mathrm{chem})}\{n_i(\mathbf{r}),\, n_e(\mathbf{r}),\, \{n_{jk}(\mathbf{x},\mathbf{y})\}\; T\} = 0$$

(j,k = electrons, ions)

$\varphi_G(\mathbf{r})$, $\varphi_C(\mathbf{r})$ *and* $\varphi_E(\mathbf{r})$ – *gravitational, centrifugal and electrostatic potentials*

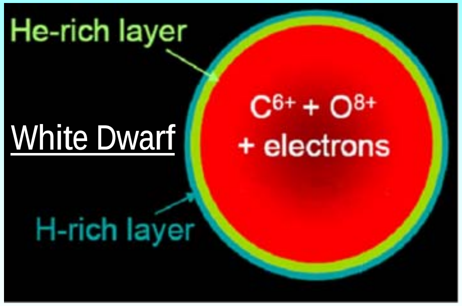
Polarization field should be equal to zero in the case of the rotation limit when the centrifugal force is equal to the gravitational one.

“…*Что касается* ***электростатического потенциала****, то . . .* ***трудно себе вообразить какие-либо особенные его проявления****. …”* ***NN****

Observable consequences *for* plasma polarization

Two well-known examples

Accretion ➔ diffusion ➔ burning *of* hydrogen *in* outer layer *of* compact stars



Chang & Bildsten (2003) *Diffusive nuclear burning in neutron star envelopes*

Mixture ${}_{12}C^{6+}$, ${}_{16}O^{8+}$, ${}_{4}He^{2+}$

$$F_E^{(p)} = -\frac{A}{Z} F_G^{(p)} \approx -2 F_G^{(p)}$$

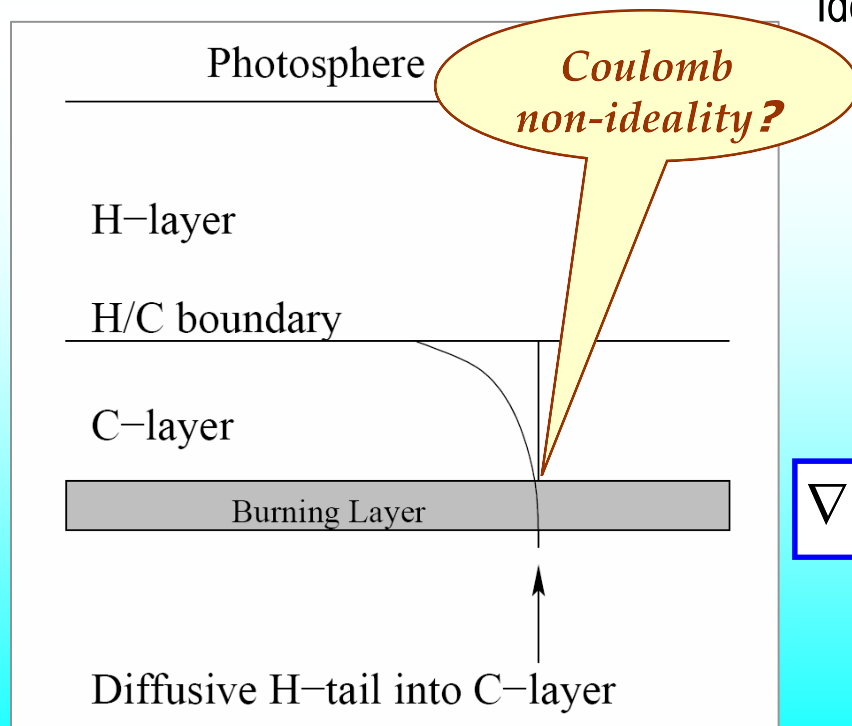
Ideal ions – degenerated electrons

$$\nabla \varphi_{\mathrm{G}}(\mathbf{r}) \qquad \nabla \varphi_{\mathrm{E}}(\mathbf{r})$$

$$F_E^{(p)} = -\frac{A}{(Z+1)} F_G^{(p)} \approx -(1.33 - 1.8) F_G^{(p)}$$

Ideal ions – non-degenerated electrons

$$\nabla \varphi_{\mathrm{G}}(\mathbf{r}) \qquad \nabla \varphi_{\mathrm{E}}(\mathbf{r})$$



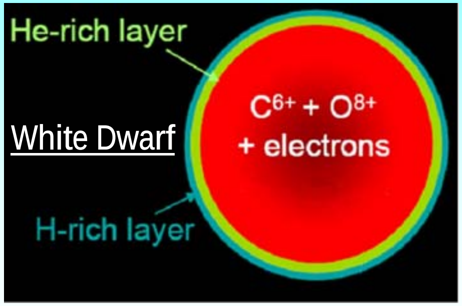
Pure hydrogen

$$F_E^{(p)} \approx -(1/2) F_G^{(p)}$$

Two well-known examples

Diffusion *and* **sedimentation** *of* **Ne** *in* **interior** *of* **WD**

Bildsten & Hall (2001) *Gravitational settling of ^{22}Ne in liquid white dwarf interior*



Mixture ${}_{12}\mathrm{C}^{6+}$, ${}_{16}\mathrm{O}^{8+}$, ${}_{4}\mathrm{He}^{2+}$ → $F_E^{(p)} = -\frac{A}{Z} F_G^{(p)} \approx -2F_G^{(p)}$

The net force on ^{22}Ne

$$\boldsymbol{F} = -22 m_p g \hat{\boldsymbol{r}} + 10 e E \hat{\boldsymbol{r}} = -2 m_p g \hat{\boldsymbol{r}}$$

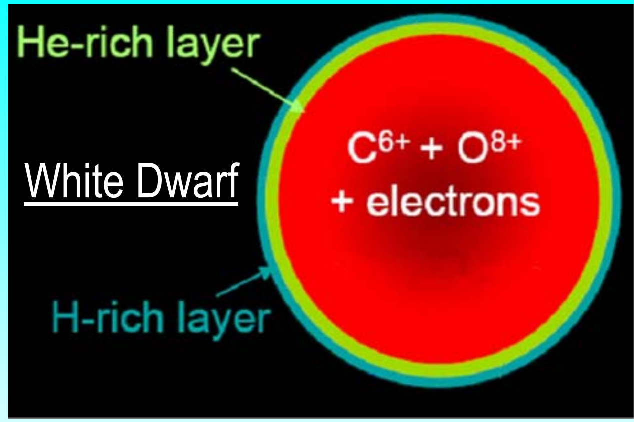
.... The total increase in cooling age by the time the WD completely crystallizes ranges from 0.25-1.6 Gyr, depending on the value of D and the WD mass.

NB !
Coulomb non-ideality at ***micro-level*** discriminates ${}_{16}\mathrm{O}^{8+}$ *in* ${}_{12}\mathrm{C}^{6+}$, *and* ${}_{12}\mathrm{C}^{6+}$ *in* ${}_{4}\mathrm{He}^{2+}$...
and accelerates Rayleigh–Taylor hydrodynamic instability

Coulomb non-ideality effect at ***macro-level*** (plasma polarization) ***suppresses*** Rayleigh–Taylor hydrodynamic instability

"…*Что касается* ***электростатического потенциала****, то . . .* ***трудно себе вообразить какие-либо особенные его проявления****. …"* *NN**

Plasma polarization *&* hydrodynamics in compact stars

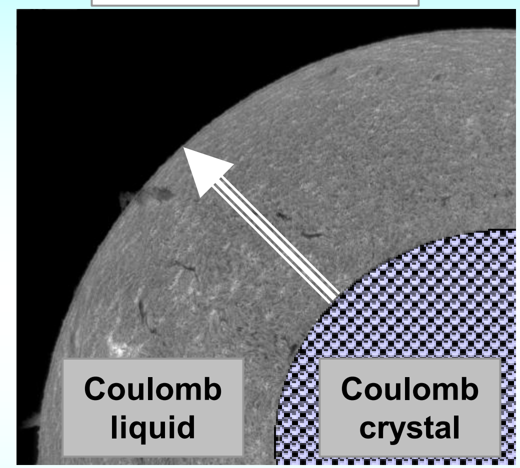


White Dwarf

White Dwarfs (WD) – is a star with mass of the Sun and size of the Earth

$M \sim 0,6 \div 1,4 M_{\odot}$

Typical WD ⇔ mixture $_{12}\mathbf{C}^{6+}$, $_{16}\mathbf{O}^{8+}$, $_{4}\mathbf{He}^{2+}$ +
+ electronic background (*strongly degenerated*)



WD – is **isothermal** approximately (*T ~ const.*)

WD – **crystallizes** during its cooling (~ 10 billions y.)
(*presumably ⇔ from the center to periphery*)

WD – is **strongly non-ideal** ($\Gamma \sim 10^2 - 10^3 >> 1$)

$T \sim 10^6 \div 10^7\,\mathrm{K}$ $\rho \sim 10^6\, g/cm^3$

$n_c \sim 3\cdot 10^{29} \div 3\cdot 10^{32}\, cm^{-3}$

$\zeta_e \equiv n_e \lambda_e^3 \sim 10^5$

$$Z_i e \nabla \varphi_E(\mathbf{r}) = -m_j \nabla \varphi_G(\mathbf{r}) \left[1 + \frac{(\mu_{ii}^0 + \Delta_i^i + Z\Delta_e^i)}{Z(Z\mu_{ee}^0 + Z\Delta_e^e + \Delta_i^e)} \right]^{-1}$$

$$x_c(\zeta_e) \equiv \left(\frac{\mu_{ii}^0}{Z\mu_{ee}^0} \right) \sim 10^{-3} \div 3\cdot 10^{-4}$$

$$\boxed{F_{\mathrm{E}}^{(\mathrm{Z})} \approx -F_{\mathrm{G}}^{(\mathrm{Z})} \left[1 - \frac{a_{\mathrm{M}}\Gamma_Z}{Z} x_c(\zeta_e) \right]^{-1} \approx -F_{\mathrm{G}}^{(\mathrm{Z})}}$$

$$\Gamma \equiv Z^2 e^2 (4\pi n_i/3)^{1/3}/kT \sim 100 \div 1000$$

Plasma polarization in interior of White Dwarfs

(*Hydrodynamics & Thermodynamics*)

Electronic subsystem is practically ***non-compressible*** due to its high degeneracy **!**

Plasma polarization in WD is close to its *zero-order* term in expansion on x_c

x_c – is ratio of electronic ideal-gas compressibility to the ionic one : $x_c \equiv (\partial\mu^0_i/\partial n_i)\,/Z(\partial\mu^0_e/\partial n_e) \ll 1$

$$F_E^{(Z)} + F_G^{(Z)} \approx 0$$

Total force acting on every ion (nuclei: $_{12}C^{6+}$, $_{16}O^{8+}$, $_{4}He^{2+}$)
is ***equal to zero*** **!**

Electrical field compensates gravitational and non-ideal forces almost totally

NB !

White Dwarf is in **weightless state** in fact **!**

What does it mean – hydrodynamics of a star in weightless state ?

Hydrodynamics of a star in weightless state ?

$$\boxed{F_E^{(Z)} + F_G^{(Z)} \approx 0}$$

Carbon, oxygen and helium does **not sink** or **float** in each other !

Any hypothetical **layered structure** from $_{12}C^{6+}$, $_{16}O^{8+}$, $_{4}He^{2+}$ is **hydrodynamically stable** as well as homogeneous mixture

Rayleigh-Taylor **hydrodynamic instability** «**does not work**» in WD !

R-T instability comes out of sources, which induce convection in WD !

NB !

Plasma polarization due to gravitation and non-ideality can **suppress hydrodynamic instability** in interiors of compact stars **!**

Crystallization on C/O mixture in White Dwarfs

Phase diagram in C/O mixture

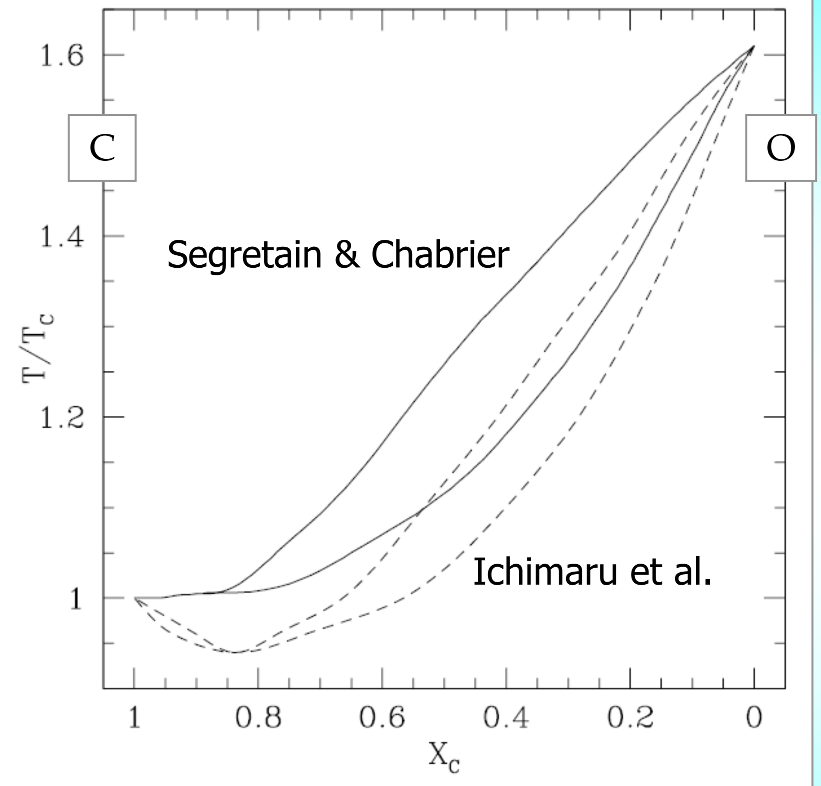


FIG. 1.—Phase diagrams for a C/O mixture as computed by Ichimaru et al. (1988, *dashed line*) and Segretain & Chabrier (1993, *solid line*), where

Phase diagram in C/O mixture

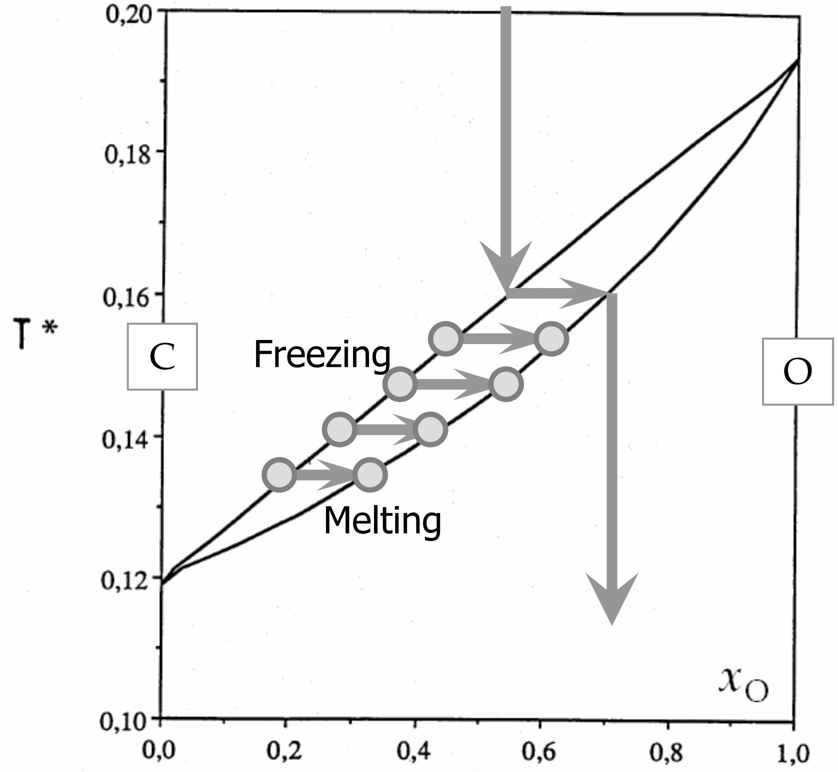
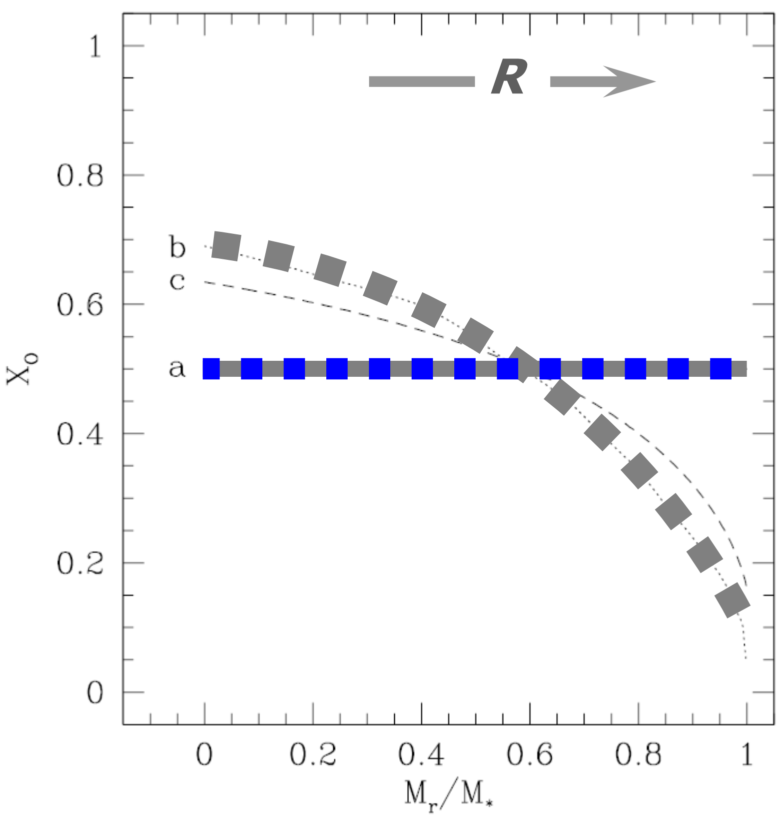


Fig. 1. Phase diagram of the carbon-oxygen mixture at constant electronic pressure. $T^* = 1/\Gamma$ is the reduced temperature,

J.Barrat, J.P.Hansen, R.Mochkovich (1988)

Crystallization on C/O mixture in White Dwarfs

Oxygen profile in WD



a) – initial
b) – final (Ichimaru)
c) – final (Segretain & Chabrier)

Phase diagram in C/O mixture

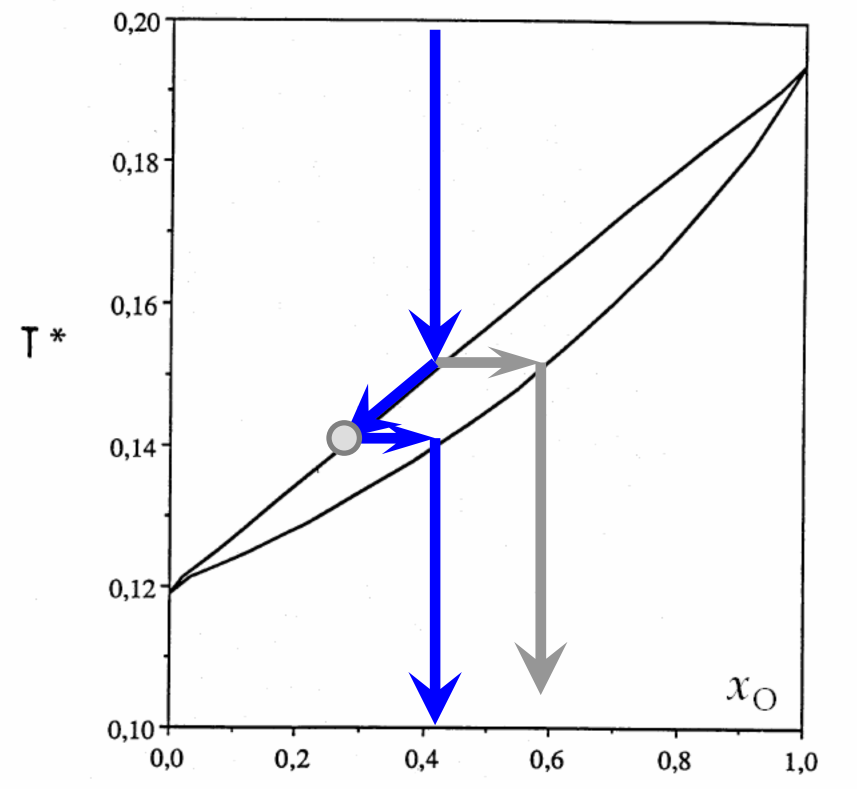


Fig. 1. Phase diagram of the carbon-oxygen mixture at constant electronic pressure. $T^* = 1/\Gamma$ is the reduced temperature,

J.Barrat, J.P.Hansen, R.Mochkovich (1988)

“…Что касается электростатического потенциала, то … трудно себе вообразить какие-либо особенные его проявления…” *NN**

Given:

Total force acting on every ion (nuclei: $_{12}C^{6+}$, $_{16}O^{8+}$, $_{4}He^{2+}$) is ~ ***equal to zero*** **!**

$$F_E^{(Z)} + F_G^{(Z)} \approx 0$$

“Naive” questions *

Why compact stars are spherical ?

Why rotating stars are spherical ? (*pancake ? roll ? more complicated ?)*

Why rotating binaries are spherical ?

What is the form of mergers (*if polarization field is taken into account*) **?**

Are all these questions meaningful ?

**(в помощь лектору по астрофизике)*

"…Что касается электростатического потенциала, то . . . трудно себе вообразить какие-либо особенные его проявления. …" *NN**

Naive questions II

What is the orientation of "Pasta" plasma ?

Structured Mixed Phase ⇔ "Pasta" plasma

'Pasta' plasma – hadron-quark phase transition in interior of neutron stars
('Mixed phase' of Glendenning *et al.*)

- Charged quark droplets (rods, slabs) in equilibrium hadron matter
- Charged hadron bubbles (tubes, slabs) in equilibrium quark matter

"Pasta" plasma

"Pasta" plasma

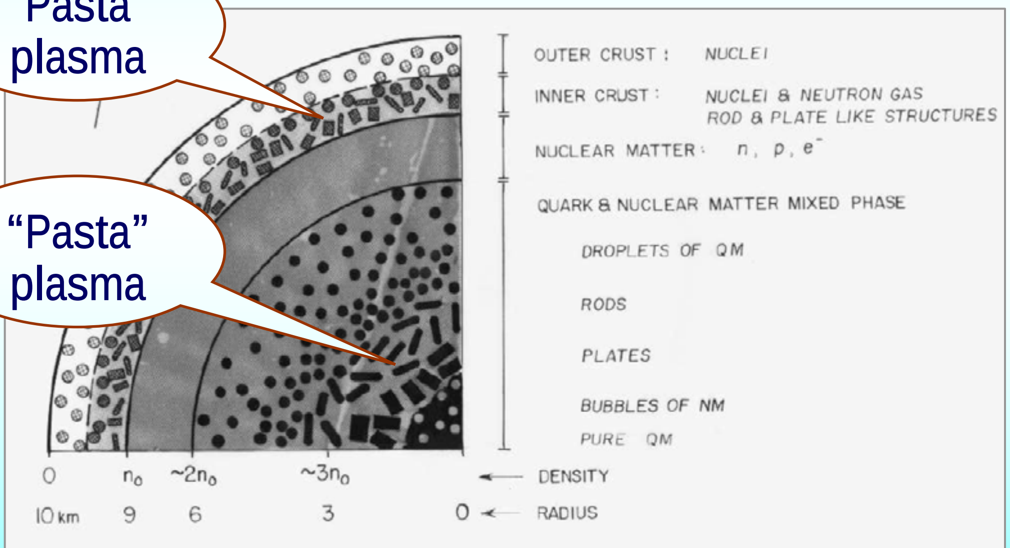
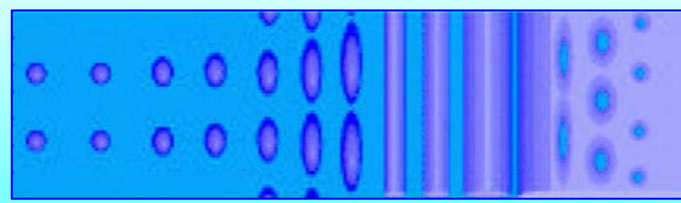


Fig. 1. Nuclear and quark matter tructures in a $\sim 1.4 M_\odot$ neutron star. Typical sizes of structures are $\sim 10^{-14} m$ but have been scaled up to be seen.

Ravenhall D., Pethick C. & Wilson J.
PRL **50** (1983)

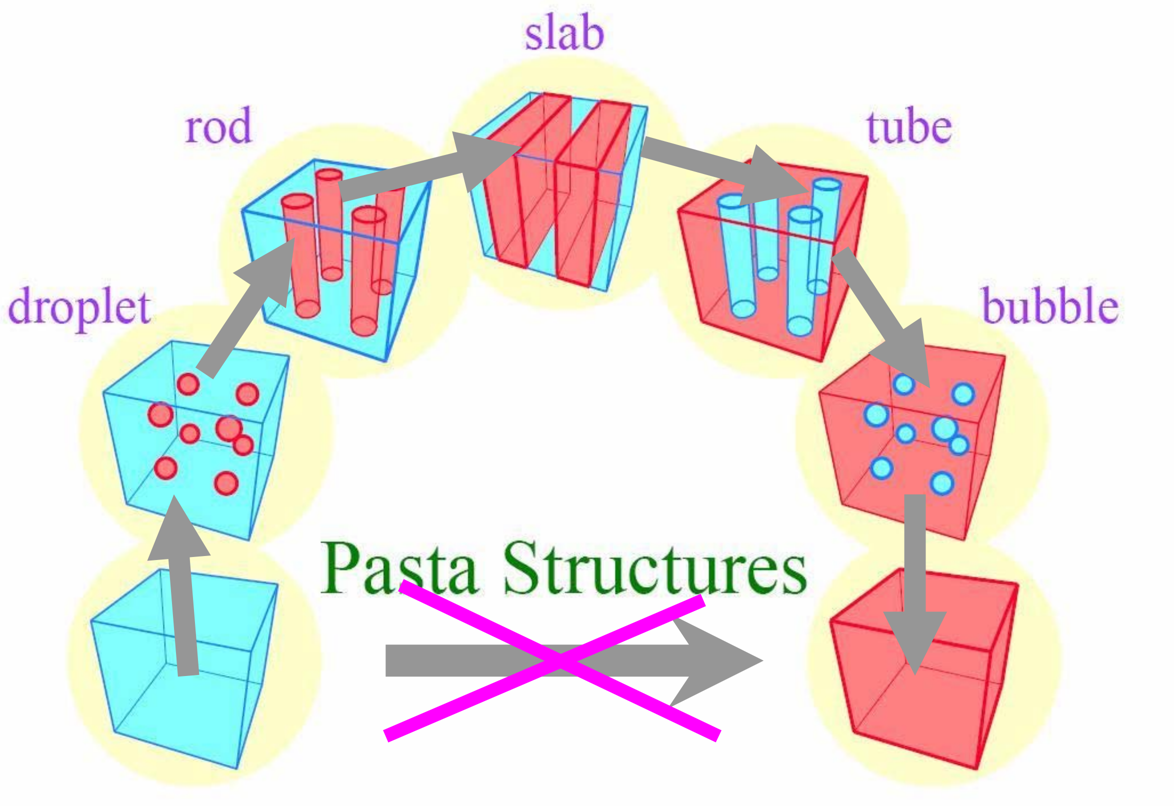
Heiselberg *and* Hjorth-Jensen
Phase Transitions in Neutron Stars
arXiv/9802028v1 (1998)

T.Maruyama, T.Tatsumi, T.Endo, S.Chiba
Pasta structures in compact stars
arXiv/0605075v2 31 (2006)



"Pasta" plasma:- *"Spaghetti" phase, "Lasagne" phase* . . . . .

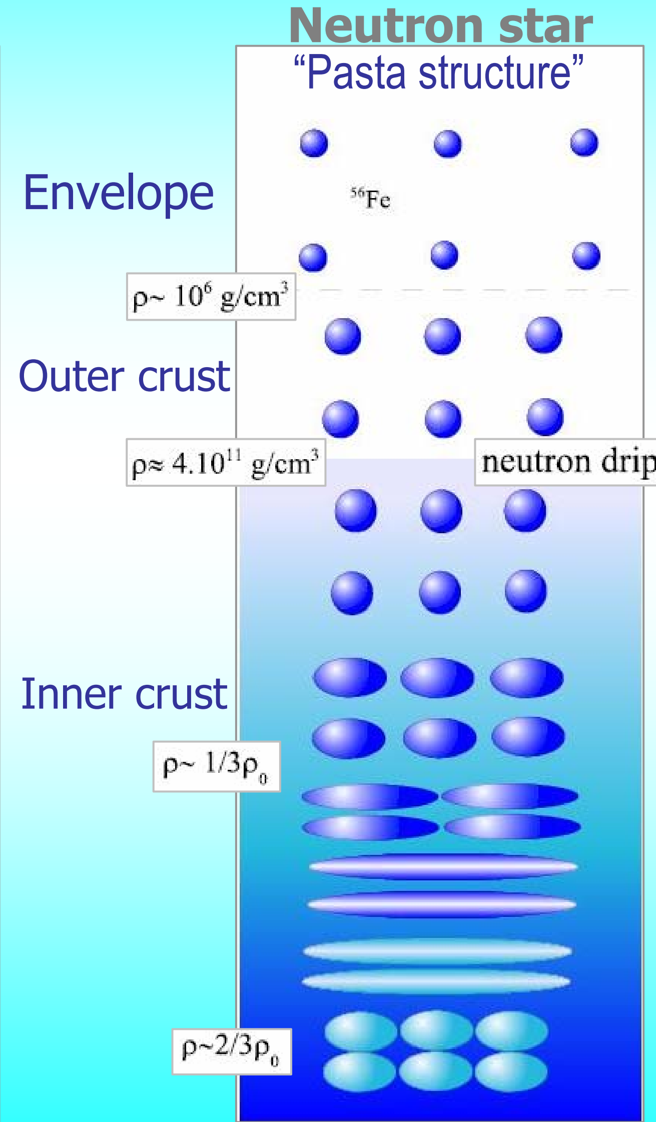
Structured Mixed Phase Concept ⇔ "Pasta"



Schematic picture of pasta structures. Phase transition from blue phase (left-bottom) to red phase (right-bottom) is considered.

Pasta structures in compact stars /arXiv:nucl-th/0605075v2 /2006/

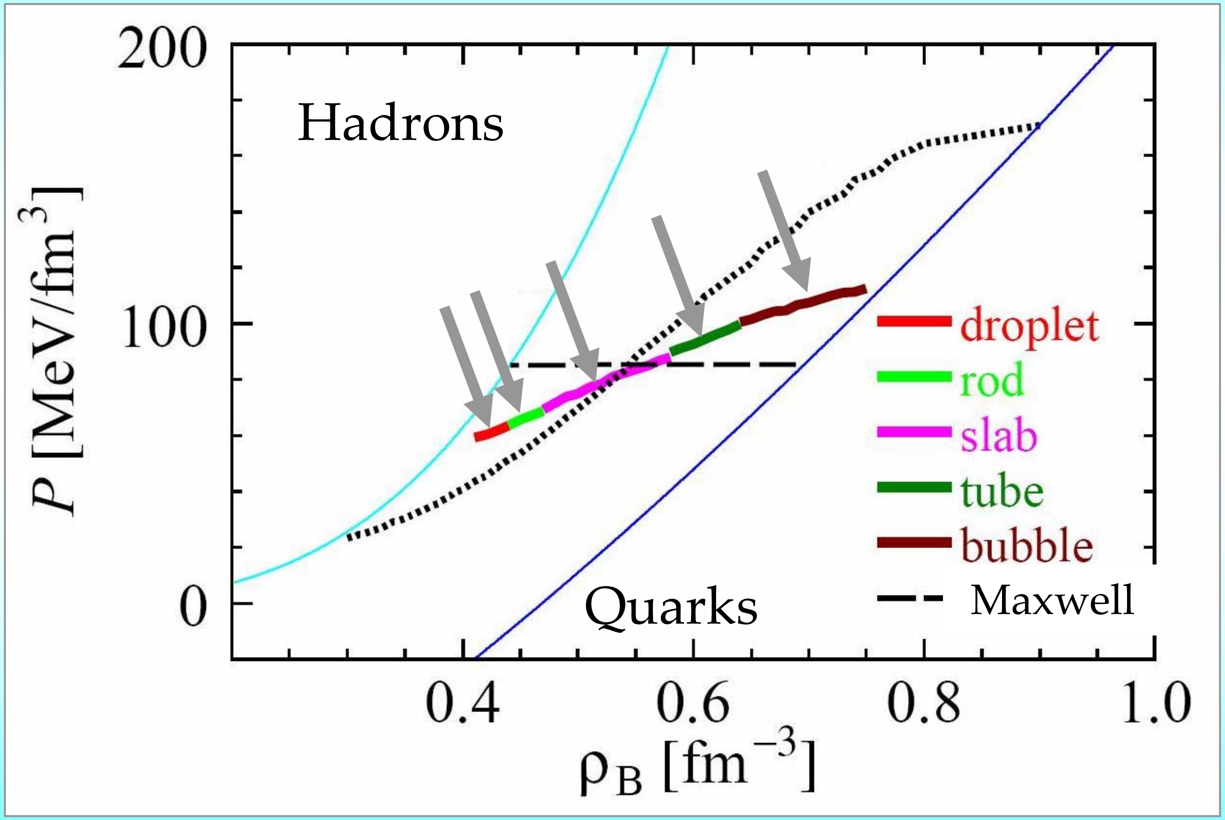
Maruyama T., Tatsumi T., Endo T., Chiba S.



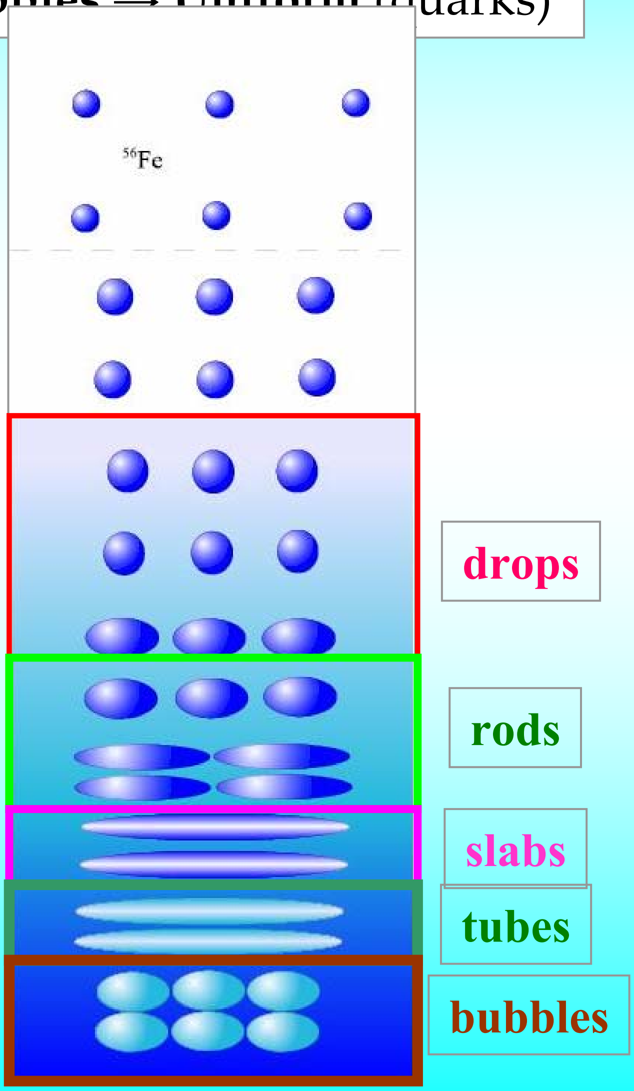
Structured Mixed Phase Concept ⇔ "Pasta"

The sequence of five (or more ?) phase transitions !

Uniform (nucleons) → **Drops** → **Rods** → **Slabs** → **Bubbles** → **Uniform** (quarks)



Maryuama T., Tatsumi T., Endo T., Chiba S.
arXiv/0605075v2



Structured Mixed Phase ⇔ "Pasta" plasma

'Pasta' pl

Uniform-I → Drops → Rods → Slabs → Bubbles → Uniform-II

– Charged quark
– Charged hadron

"Pasta" plasma

"Pasta" plasma

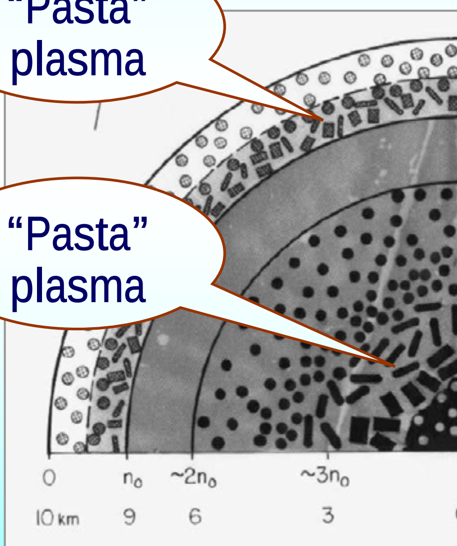


Fig. 1. Nuclear and quark matter t
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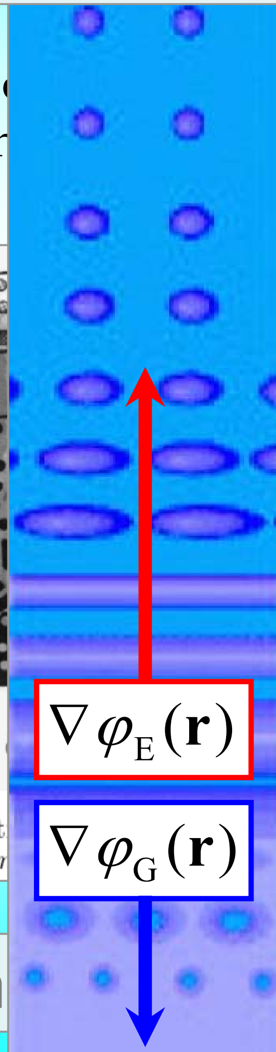


What *is the* **orientation** *of* **spaghetti** *and* **lasagne ?**

Heise ... Hjorth-Jensen
Phase T ... s in Neutron Stars
a ... 028v1 (1998)

?

What *is the* **topology** *(connectivity) of* **spaghetti** *and* **lasagne ?**

Honeycomb ?

What *is the* **thermoconductivity** *of such* **mist-net-foam** *structure* **?**

"Pasta" plasma

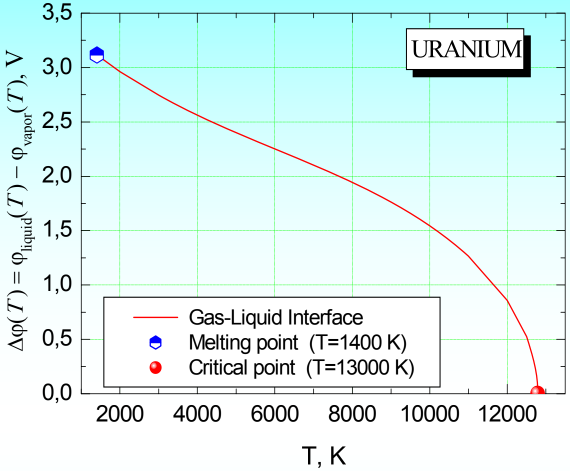
*"…Что касается **электростатического потенциала**, то . . . **трудно себе вообразить какие-либо особенные его проявления**. …"* ***NN****

Macroscopic charge *on* **phase boundaries** *in MAO*

Electrostatics of phase boundaries in Coulomb systems

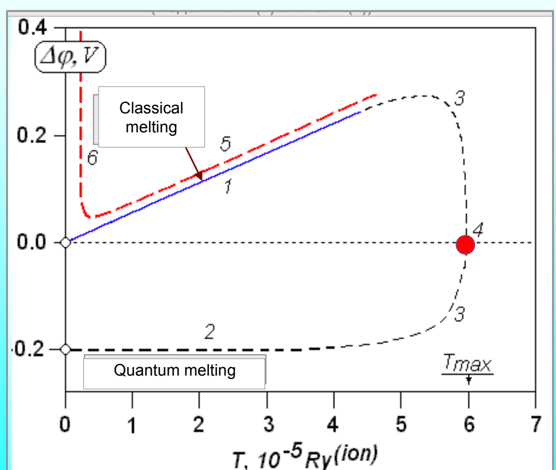
Terrestrial applications

Electrostatic (Galvani) potential



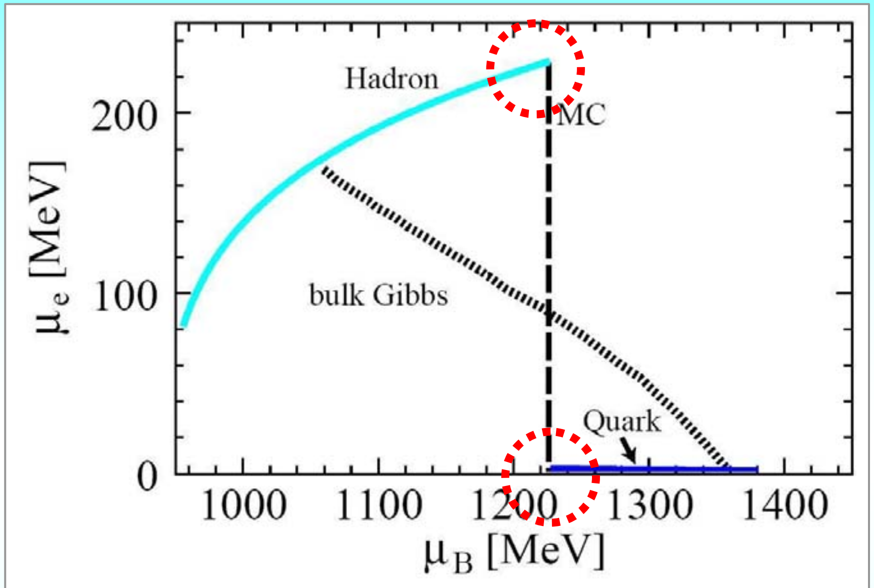
Iosilevskiy & Gryaznov, *J.Nucl.Mat.* (2005)

Electrostatic "portrait" of Wigner crystal in OCP



Iosilevskiy & Chigvintsev, *J. Physique* (2000)

Quark-Hadron phase transition in NS



Phase diagram in the $\mu_B - \mu_e$ plane.

Endo T., Maruyama T., Chiba S., Tatsumi T. arXiv:astro-ph/0601017v1/ 2006 /

$$e\Delta\varphi_{HQ} = (\mu_e)_{\rm Hardron\ phase} - (\mu_e)_{\rm Quark\ phase}$$

$$e\Delta\varphi_{HQ} \approx 200\ \text{MeV}\ \ !$$

$$\delta_{HQ} \approx 10^3\ \text{fm} \ \rightarrow\ E \sim 10^{18}\ \text{V/cm}$$

For comparison: Alcock *et al.*, 1986: $\rightarrow$ $E \sim \mathbf{10^{17}}$ V/cm

Macroscopic charge *on* **phase boundaries**
in massive astrophysical objects

$$\boxed{e\nabla\varphi_{\mathrm{E}}(\mathbf{r}) = -\nabla\varphi_{\mathrm{G}}(\mathbf{r})\frac{\left\langle \mathrm{Z}\middle|\mathbf{D}_{\mu}^{n}\middle|\mathrm{M}\right\rangle}{\left\langle \mathrm{Z}\middle|\mathbf{D}_{\mu}^{n}\middle|\mathrm{Z}\right\rangle}}$$

Basic statement:

Any **jump-like discontinuity** in thermodynamic parameters (**phase boundary**, **jump** in ionic **composition** *etc*) must be accompanied with existing of **macroscopic charge** localized at this interface.

astro-ph:0901.2547 / astro-ph:0902.2386

Plasma polarization in thermodynamics of neutron stars

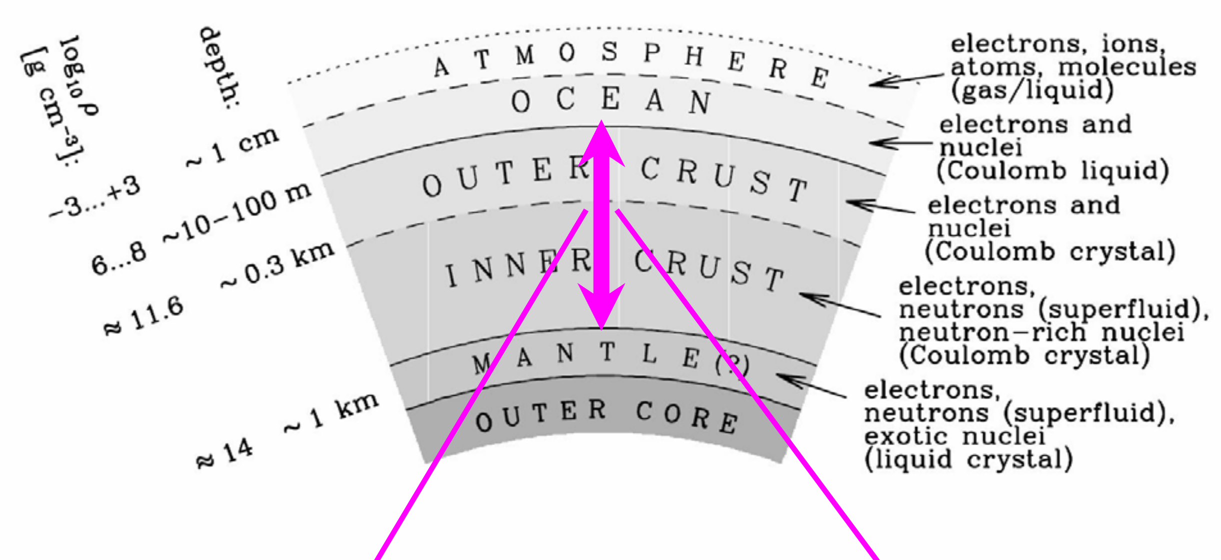
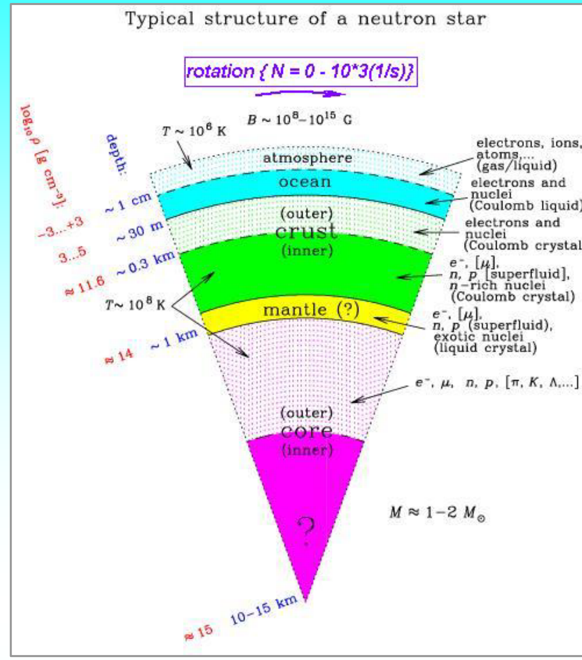
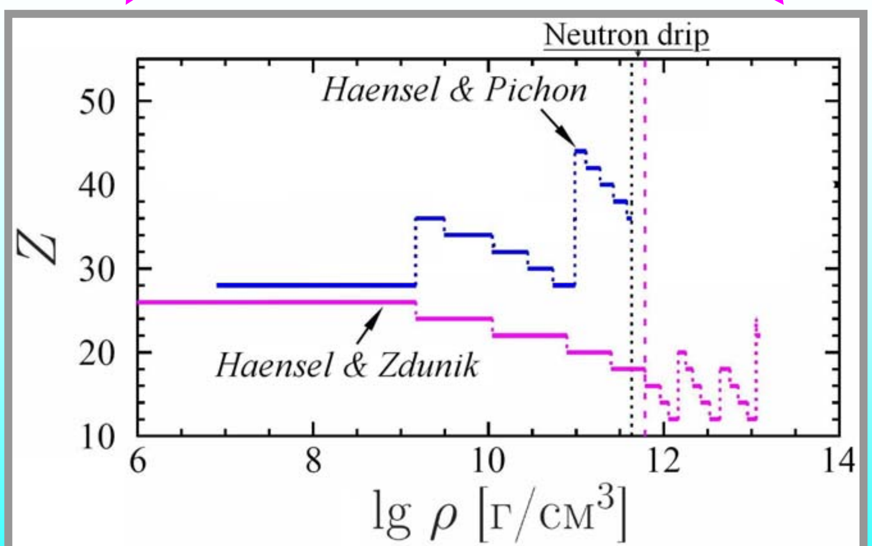


Figure 2.1. Schematic structure of an envelope of a neutron star with the internal temperature $\sim 10^8$ K (see text for more details).



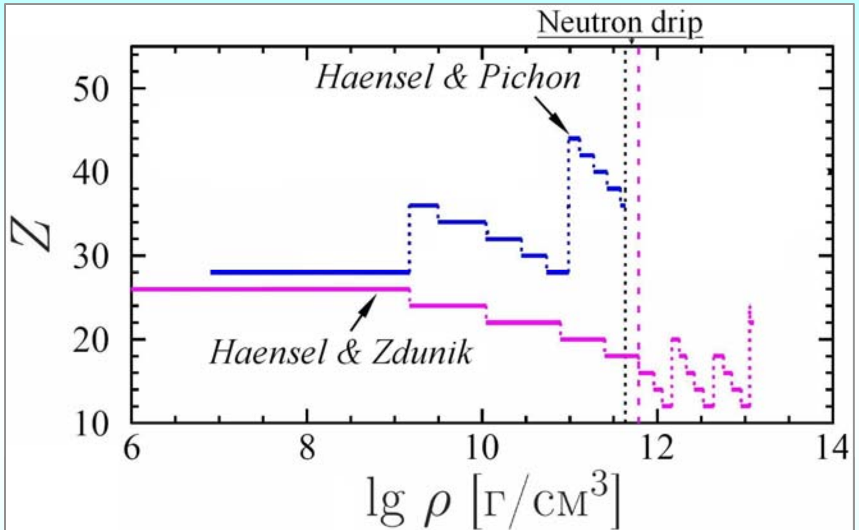


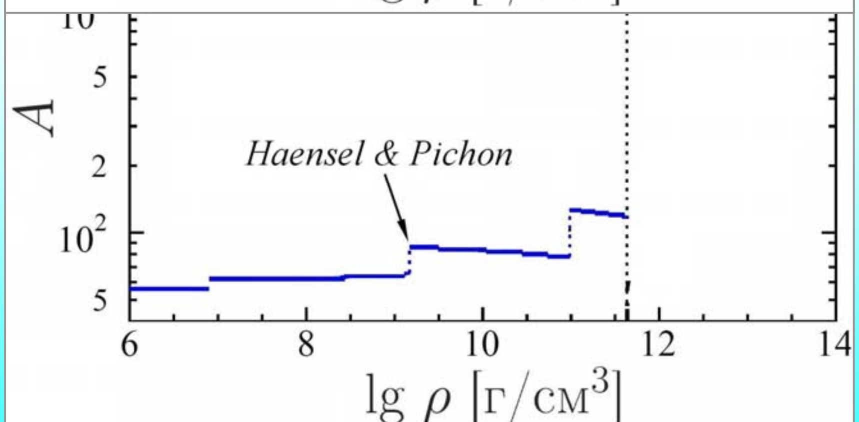
After / Haensel P, Potekhin A, Yakovlev D, *Neutron Stars* // Springer, 2007 /

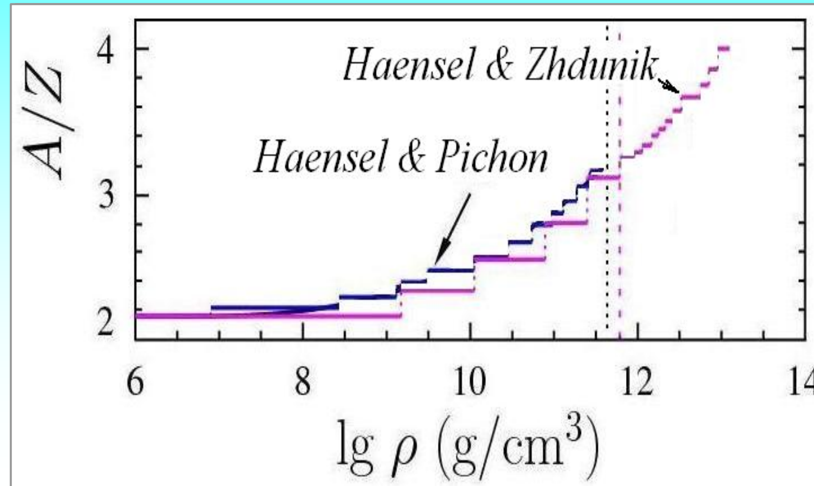
Macroscopic charge on phase boundaries in MAO

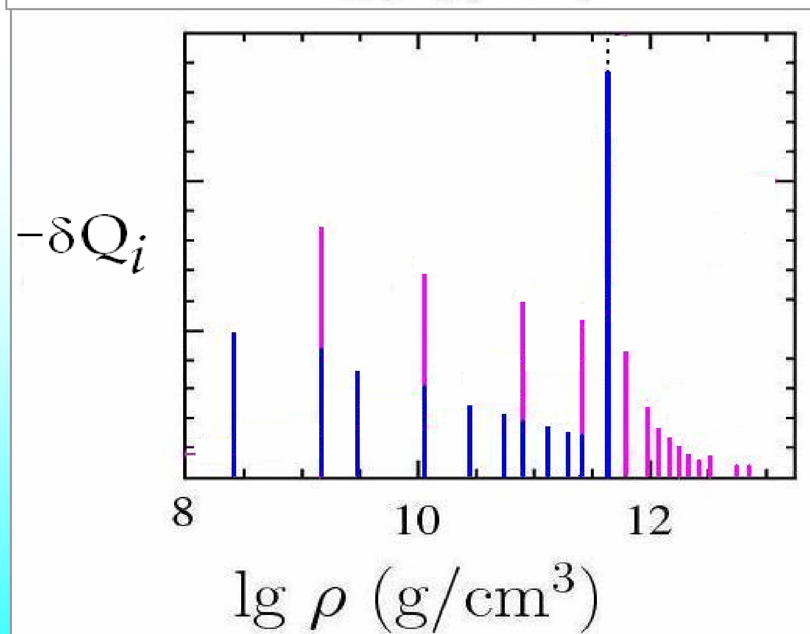
Typically – ratio ***A/Z increases*** when we cross the interface toward the inner layer.
It means ***decreasing*** of electrostatic field, i.e. macroscopic ***negative charge*** localized on two-layer interface.

$$e\nabla\varphi_{\mathrm{E}}(\mathbf{r}) = -\nabla\varphi_{\mathrm{G}}(\mathbf{r})\frac{\langle \mathrm{Z}|\mathbf{D}_{\mu}^{n}|\mathrm{M}\rangle}{\langle \mathrm{Z}|\mathbf{D}_{\mu}^{n}|\mathrm{Z}\rangle} \cong -m_{p}\nabla\varphi_{\mathrm{G}}(\mathbf{r})\frac{A}{Z}$$









Conclusions and perspectives

- **Plasma polarization** in massive astrophysical bodies is **general** phenomenon

- **Plasma polarization** in massive astrophysical bodies is **universal** phenomenon

- **Plasma polarization** in massive astrophysical bodies is **interesting** phenomenon

- **Plasma polarization** in massive astrophysical bodies manifests itself in great number of **observable consequences** in **thermodynamics** of MAO

- **Plasma polarization** in massive astrophysical bodies manifests itself in great number of **observable consequences** in **hydrodynamics** of MAO

- **Coulomb non-ideality** effects at **micro**-level could **amplify hydrodynamic instability** in MAO, while **Coulomb non-ideality** at **macro**-level could **suppress hydrodynamic instability**

Theses

"Теория строения белых карликов сравнительно проста, хорошо разработана и согласуется с наблюдениями…"

*N**

"…Что касается **электростатического потенциала**, *то уделять ему особое внимание не представляется необходимым, потому что* **трудно себе вообразить какие-либо особенные его проявления**. *…"* *NN**

"… Возможно это правда, что … среднего электрического поля получила недостаточно внима… …роны астрофизиков, но кажется, что это как раз и обусловлено… …ием его какой-либо роли…

…… Было бы очень инте… …идеть какие-либо наблюдательные последствия … …ектрического поля …"

*NN**

?

<u>Об электризации, вызванной тяготением массивного тела</u>

"…Из-за малости параметра $\alpha = Gm_p^2/e^2$ перечисленные величины исключительно малы, и ***рассматриваемый эффект не может иметь прямых наблюдательных последствий****…"*

*NNN**

Thank you!

Support: **ISTC** 3755 // **CRDF** MO-011-0, and by **RAS Scientific Programs**
"Physics and Chemistry of Extreme States of Matter" and "Physics of Compressed Matter and Interiors of Planets"
MIPT Education Center "Physics of High Energy Density Matter"