Data presented in this page are hydrogen dipole transition probabilities per time unit (the Einstein coefficients). These values are required in the investigations of astrophysics (e.g. the interstellar medium, a nebulae, the primordial recombination) and plasma physics.

The exact Einstein coefficients given via hypergeometric functions (see Berestetzkii et al) are complicated and demand much computer time to calculate. That is why several approximations have been developed (see Kaplan, Pikelner; Johnson). In our work we used exact formulae and Johnson's approximation.

The exact expression of the transition coefficient A_{if} averaged over angular momentum is:

$$A_{if} = \frac{4\omega^3}{3\hbar c^3}\left(\frac{\hbar^2}{me}\right)^2 |<f|r|i>|^2 \tag{1}$$

where i is the principal quantum number of initial state, f is principal quantum number of final state, $\omega = 2\pi\nu_{if}$ is cyclic frequency of transition $i \rightarrow f$, $\hbar$ is the Planck constant, m is the electron mass, e is the electron charge, $<f|r|i>$ is the dimensionless transition matrix element averaged over angular momentum:

$$|<f|r|i>|^2 = \frac{1}{i^2}\sum_{l_1=0}^{i-1}\sum_{l_2=0}^{f-1} |<f, l_1||r||i, l_2>|^2 \tag{2}$$

where $<f, l_1||r||i, l_2>$ is the reduced matrix element of transition $i, l_2 \rightarrow f, l_1$. These values are symmetric relative initial and final states and diiffer from zero only for transition with momentum change equal ± 1.

$$|<f, l-1||r||i, l>| = \sqrt{l}\frac{1}{4(2l-1)!}\cdot B_1 \cdot B_2 \tag{3}$$

$$B_1 = \sqrt{\frac{(i+l)!(f+l-1)!}{(i-l-1)!(f-l)!}}\frac{(4if)^{l+1}(i-f)^{i+f-2l-2}}{(i+f)^{i+f}} \tag{4}$$

$$B_2 = \left|F\left(-i+l+1, -f+l, 2l, -\frac{4if}{(i-f)^2}\right) - \left(\frac{i-f}{i+f}\right)^2 F\left(-i+l-1, -f+l, 2l, -\frac{4if}{(i-f)^2}\right)\right| \tag{5}$$

where F is the hypergeometric function:

$$F(\alpha, \beta, \gamma, z) = 1 + \frac{\alpha\beta}{\gamma}\frac{z}{1!} + \frac{\alpha(\alpha+1)\beta(\beta+1)}{\gamma(\gamma+1)}\frac{z^2}{2!} + ... \tag{6}$$

The approximation formula is:

$$A_{if} = \frac{2^6 e^2 I^2}{3^{3/2}\pi m c^3 \hbar^3}\frac{g_{bb}(i, f)}{fi^5 - f^3 i^3} \tag{7}$$

where I is the ground state ionization potential, $g_{bb}(i, f)$ is the Gaunt factor for bound-bound transition:

$$g_{bb}(i, f) = g_0(f) + g_1(f)x^{-1} + g_2(f)x^{-2} \tag{8}$$

where x is the ratio of the transition energy to the ionization energy of final state:

$$x = E_{if}/I_f = 1 - (f/i)^2 \tag{9}$$

Gaunt factor coefficients

	$f = 1$	$f = 2$	$f \geq 3$
$g_0(f)$	1.1330	1.0785	$0.9935 + 0.2328 f^{-1} - 0.1296 f^{-2}$
$g_1(f)$	−0.4059	−0.2319	$-f^{-1}(0.6282 - 0.5598 f^{-1} + 0.5299 f^{-2})$
$g_2(f)$	0.07014	0.02947	$f^{-2}(0.3887 - 1.181 f^{-1} + 1.470 f^{-2})$