Computational modelling of non-thermal plasma in solar flares

Mykola Gordovskyy

Outline

- Method classification
- Test-particle approach
- Particle-in-Cell method
- Vlasov and Vlasov-Maxwell approaches
- Hybrid methods

Classification

	Lagrangian	Eulerean

	Lagrangian	Eulerean
Non-self-consistent		
Self-consistent		

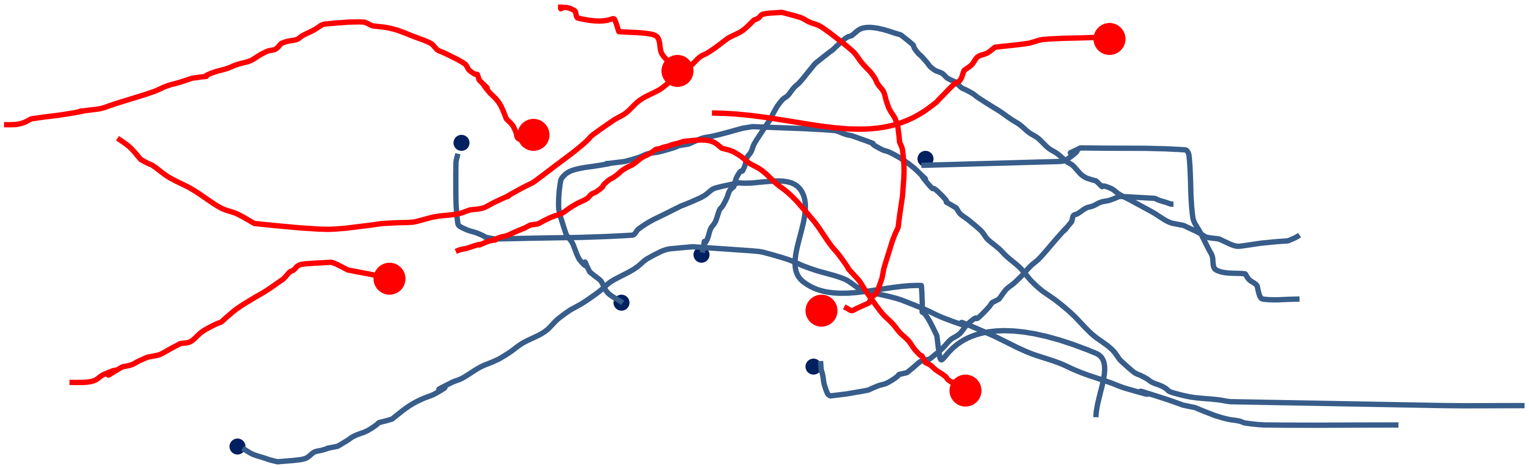
Full trajectories

Gyro-averaging

With **Without**

particle interaction

Non-self-consistent lagrangian methods



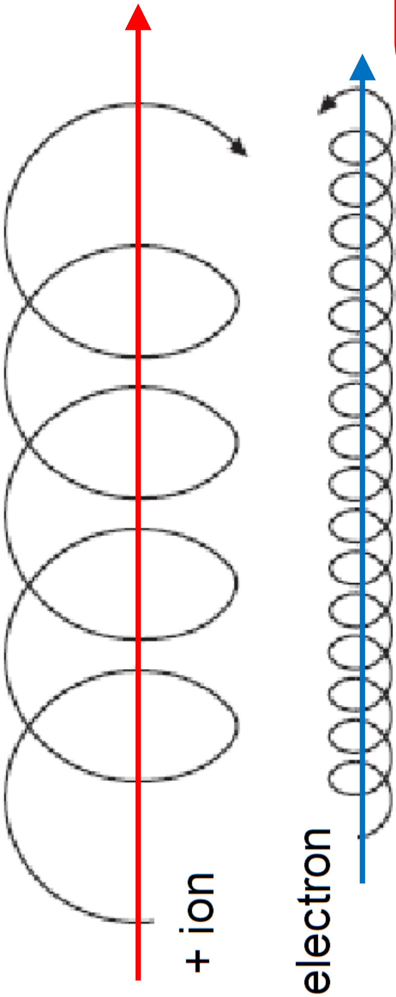
Test-particle method

Full trajectory integration

$$\frac{d\mathbf{r}}{dt} = \mathbf{V}$$

$$\frac{d\mathbf{V}}{dt} = \frac{q}{m}\left[\mathbf{E} + \mathbf{V} \times \mathbf{B}\right]$$

Non-self-consistent lagrangian methods



Guiding-centre approximation

$$\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} = \boldsymbol{u} + v_{\|}\boldsymbol{b}$$

$$\begin{aligned}\boldsymbol{u} = {} & \boldsymbol{u}_E + \frac{m}{q}\frac{v_{\|}^2}{B}[\boldsymbol{b}\times(\boldsymbol{b}\cdot\boldsymbol{\nabla})\boldsymbol{b}] + \frac{m}{q}\frac{\mu}{B}[\boldsymbol{b}\times(\boldsymbol{\nabla}B)] \\ & +\frac{m}{q}\frac{v_{\|}}{B}[\boldsymbol{b}\times(\boldsymbol{b}\cdot\boldsymbol{\nabla})\boldsymbol{u}_E] + \frac{m}{q}\frac{v_{\|}}{B}[\boldsymbol{b}\times(\boldsymbol{u}_E\cdot\boldsymbol{\nabla})\boldsymbol{b}] \\ & +\frac{m}{q}\frac{1}{B}[\boldsymbol{b}\times(\boldsymbol{u}_E\cdot\boldsymbol{\nabla})\boldsymbol{u}_E]\end{aligned}$$

$$\begin{aligned}\frac{\mathrm{d}v_{\|}}{\mathrm{d}t} = {} & \frac{q}{m}\boldsymbol{E}\cdot\boldsymbol{b} - \mu(\boldsymbol{b}\cdot\boldsymbol{\nabla}B) + v_{\|}\boldsymbol{u}_E\cdot((\boldsymbol{b}\cdot\boldsymbol{\nabla})\boldsymbol{b}) \\ & +\boldsymbol{u}_E\cdot((\boldsymbol{u}_E\cdot\boldsymbol{\nabla})\boldsymbol{b})\end{aligned}$$

$$\frac{\mathrm{d}\mu}{\mathrm{d}t} = 0.$$

$$\boldsymbol{b} = \frac{\boldsymbol{B}}{B} \qquad \boldsymbol{u}_E = \frac{\boldsymbol{E}\times\boldsymbol{b}}{B}$$

Non-self-consistent lagrangian methods

Parameter (1)	ω_B (Hz) (2)	P_B (s) (3)	$\varepsilon = 10^2$ eV: V (m s^{-1}) (4)	$\varepsilon = 10^2$ eV: R_B (m) (5)	$\varepsilon = 10^4$ eV: V (m s^{-1}) (6)	$\varepsilon = 10^4$ eV: R_B (m) (7)
Electrons:						
$B = 10^{-2}$ T	3.0×10^8	2.1×10^{-8}	10^6	5×10^{-4}	10^8	5×10^{-2}
$B = 10^{-4}$ T	3×10^6	2.1×10^{-6}	10^6	5×10^{-2}	10^8	5
Protons:						
$B = 10^{-2}$ T	1.7×10^5	3.8×10^{-5}	10^5	1	10^7	10^2
$B = 10^{-4}$ T	1.7×10^3	3.8×10^{-3}	10^5	10^2	10^7	10^4

Full trajectory

- Need to resolve Larmor radii

Guiding-centre approximation

- Larmor radii need to be much smaller than characteristic E & B scales

- Larmor periods need to be much smaller than typical E & B variation times

Non-self-consistent lagrangian methods

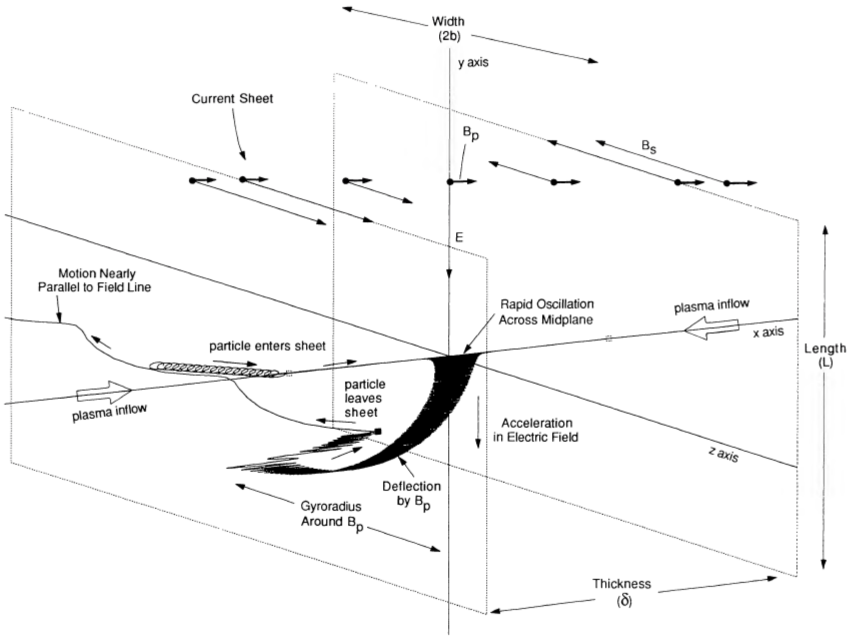


FIG. 1.—A Typical proton orbit in the current sheet of a two-ribbon flare. Inside the sheet the motion along the $\hat{y}$ and $\hat{z}$ axes is scaled down by a factor 250, respectively, 500 for clarity of presentation.

Martens & Young 1990

Zharkova & Gordovskyy 2004

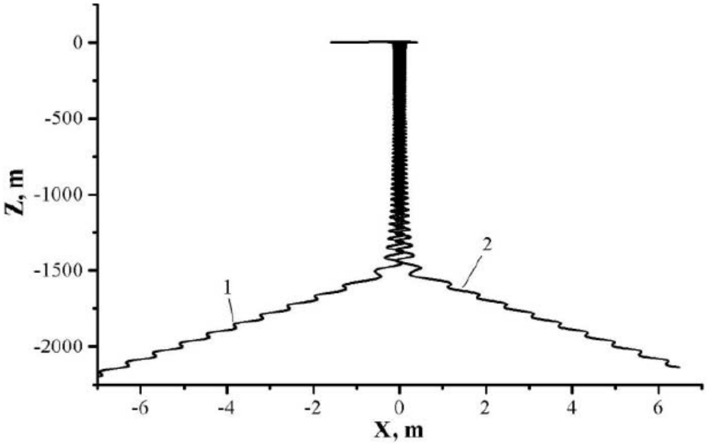


FIG. 2.—Typical trajectories of protons in the (X, Z)-plane with protons entering from the top. The label "1" corresponds to the case $B_y < 0$, while the label "2" corresponds to the case $B_y > 0$.

Non-self-consistent lagrangian methods

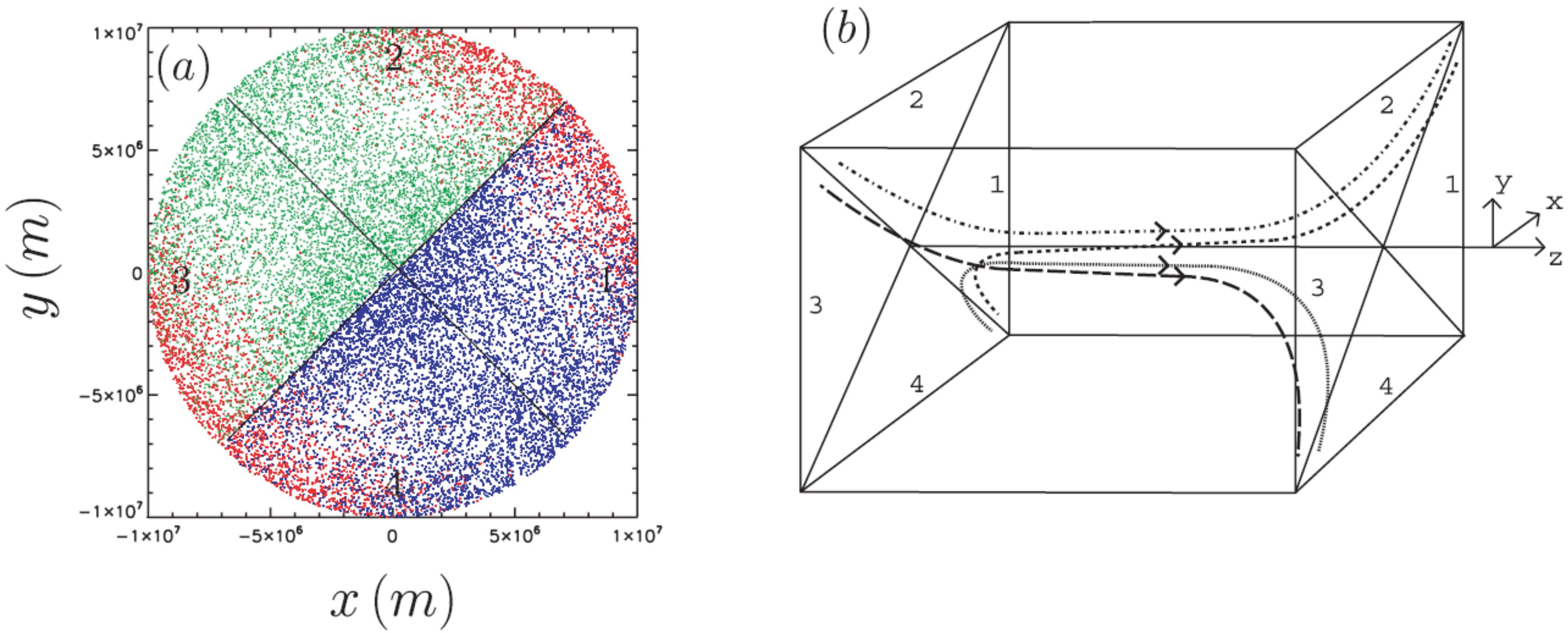
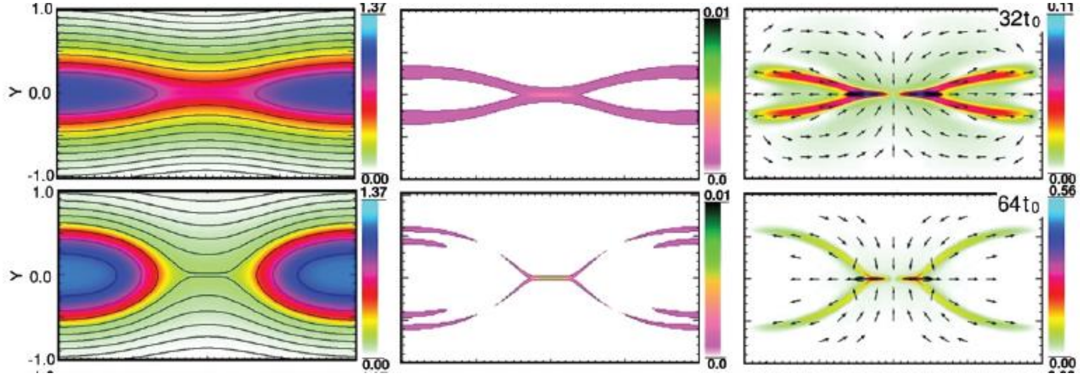
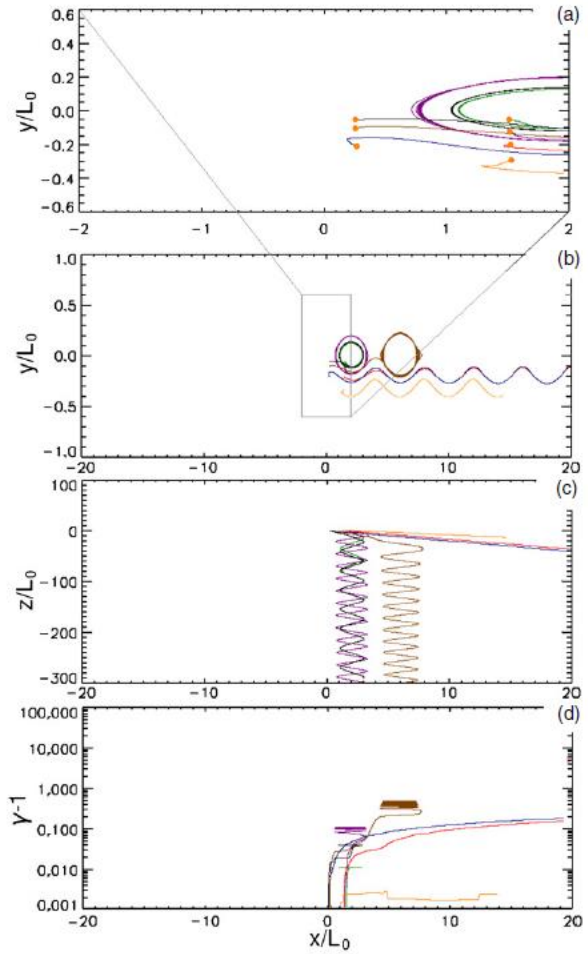


FIG. 2.—(*a*) Starting position of electrons crossing the $R = R_0$ boundary within 1 s for $B_z = 10^{-5}$ T and $\tau = 2.5 \times 10^{-2}$ s at secondary $\phi = \pi/4$ and $5\pi/4$ footpoints (*red*), $\phi = 3\pi/4$ primary footpoint (*green*), and $\phi = 7\pi/4$ primary footpoint (*blue*). (*b*) Cartoon of field lines for an X-point field with finite B_z. A positive E_z will have a component parallel to the magnetic field such that most electrons in regions 1 and 4 will be accelerated to the $\phi = 7\pi/4$ footpoint.

Hamilton et al 2005

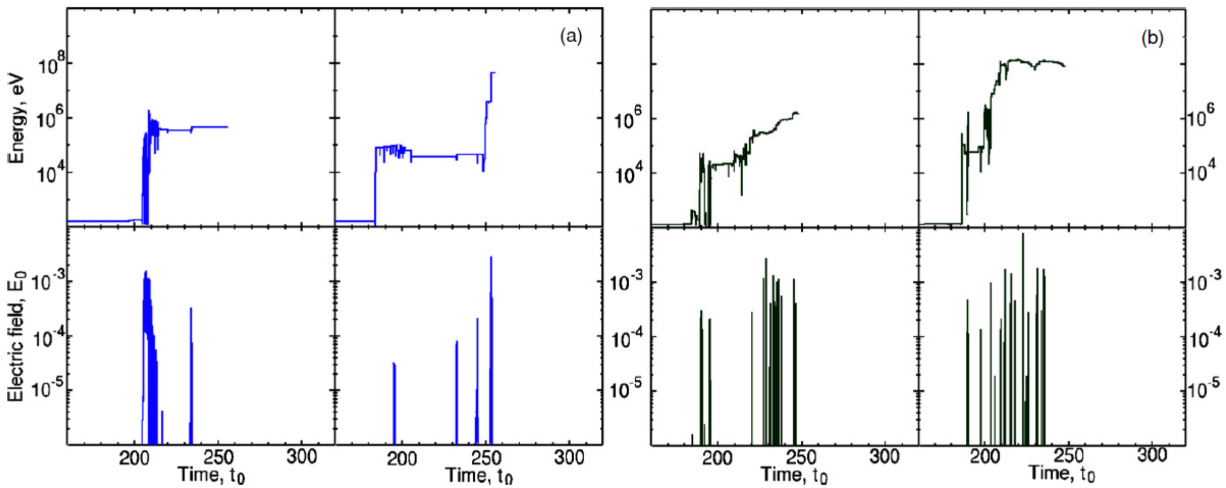
Non-self-consistent lagrangian methods





Gordovskyy et al 2010, 2011

Non-self-consistent lagrangian methods



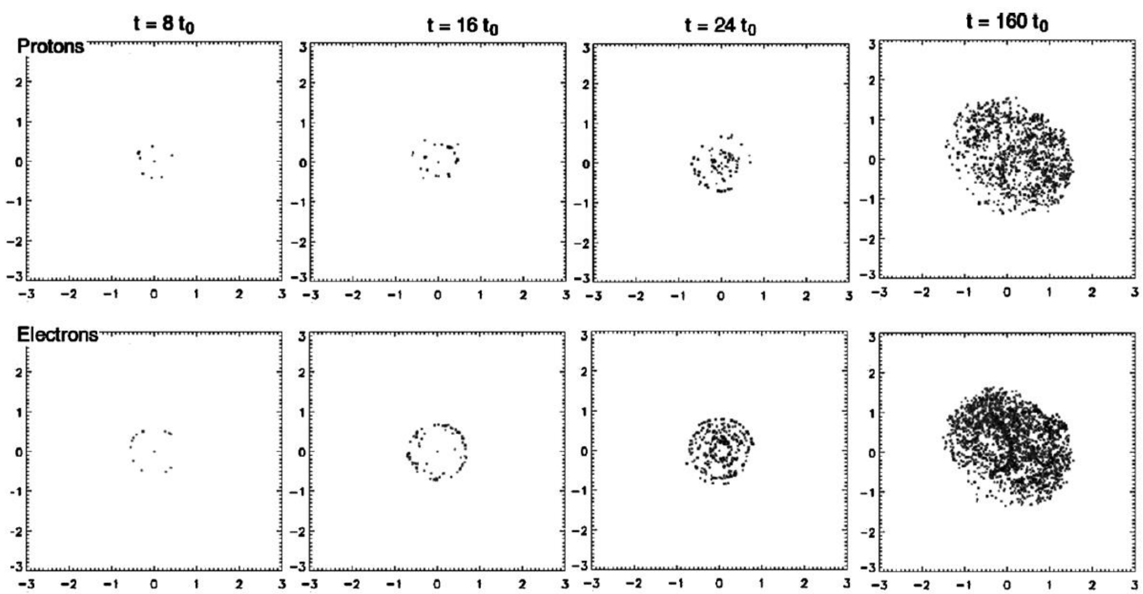


Figure 5. Distribution of protons and electrons with the energy >10 keV near the "footpoint" boundaries ($|z| > 9.5\, L_0$).

Gordovskyy et al 2014

Non-self-consistent lagrangian methods

Non-self-consistency problem

- Method is valid if $E_{part} << E_{sys}$ and $j_{part} << j_{sys}$

Undersampling problem

- Limited number of test-particles causes problems when the distribution function is small

Non-self-consistent lagrangian methods

Test-particles with collisions

$$\frac{d\boldsymbol{r}}{dt} = \boldsymbol{u} + \frac{(\gamma v_{||})}{\gamma}\boldsymbol{b}$$

$$\boldsymbol{u} = \boldsymbol{u}_E + \frac{m}{q}\frac{(\gamma v_{||})^2}{\gamma\kappa^2 B}[\boldsymbol{b}\times(\boldsymbol{b}\cdot\boldsymbol{\nabla})\boldsymbol{b}] + \frac{m}{q}\frac{\mu}{\gamma\kappa^2 B}[\boldsymbol{b}\times\boldsymbol{\nabla}(\kappa B)]$$

$$\frac{d(\gamma v_{||})}{dt} = \frac{q}{m}\boldsymbol{E}\cdot\boldsymbol{b} - \frac{\mu}{\gamma}(\boldsymbol{b}\cdot\boldsymbol{\nabla}(\kappa B)) + \left[v\frac{\delta\alpha}{\delta t}\right]_{coll} + \left[\alpha\frac{\delta v}{\delta t}\right]_{coll}$$

$$\frac{d\mu}{dt} = \left[\frac{v(1-\alpha^2)}{B}\frac{\delta v}{\delta t}\right]_{coll} - \left[\frac{\alpha v^2}{B}\frac{\delta\alpha}{\delta t}\right]_{coll}.$$

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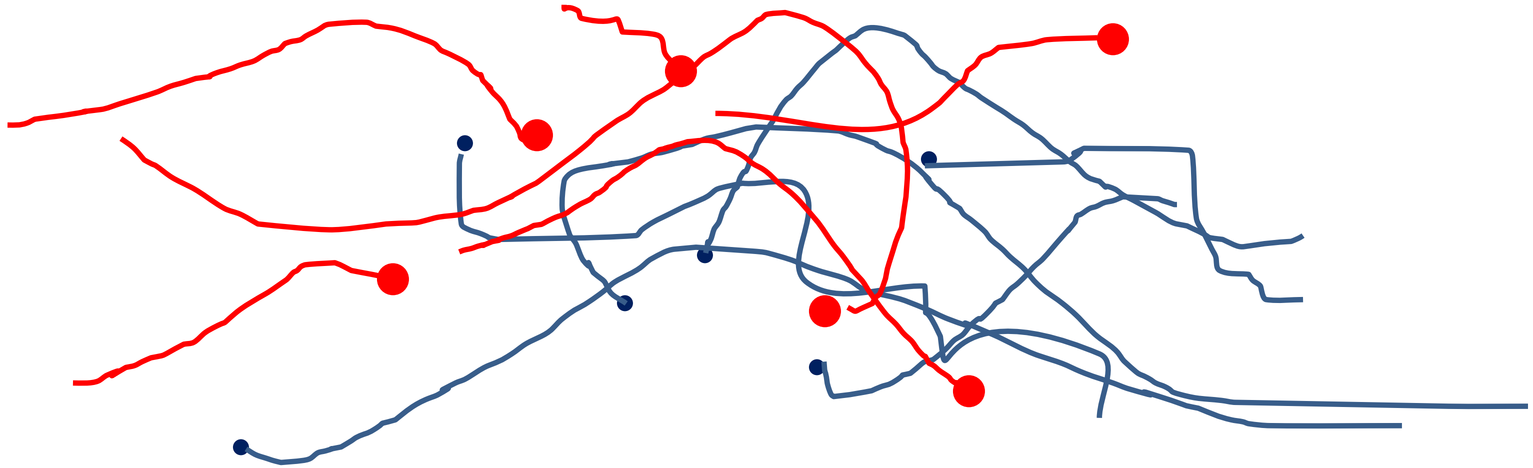
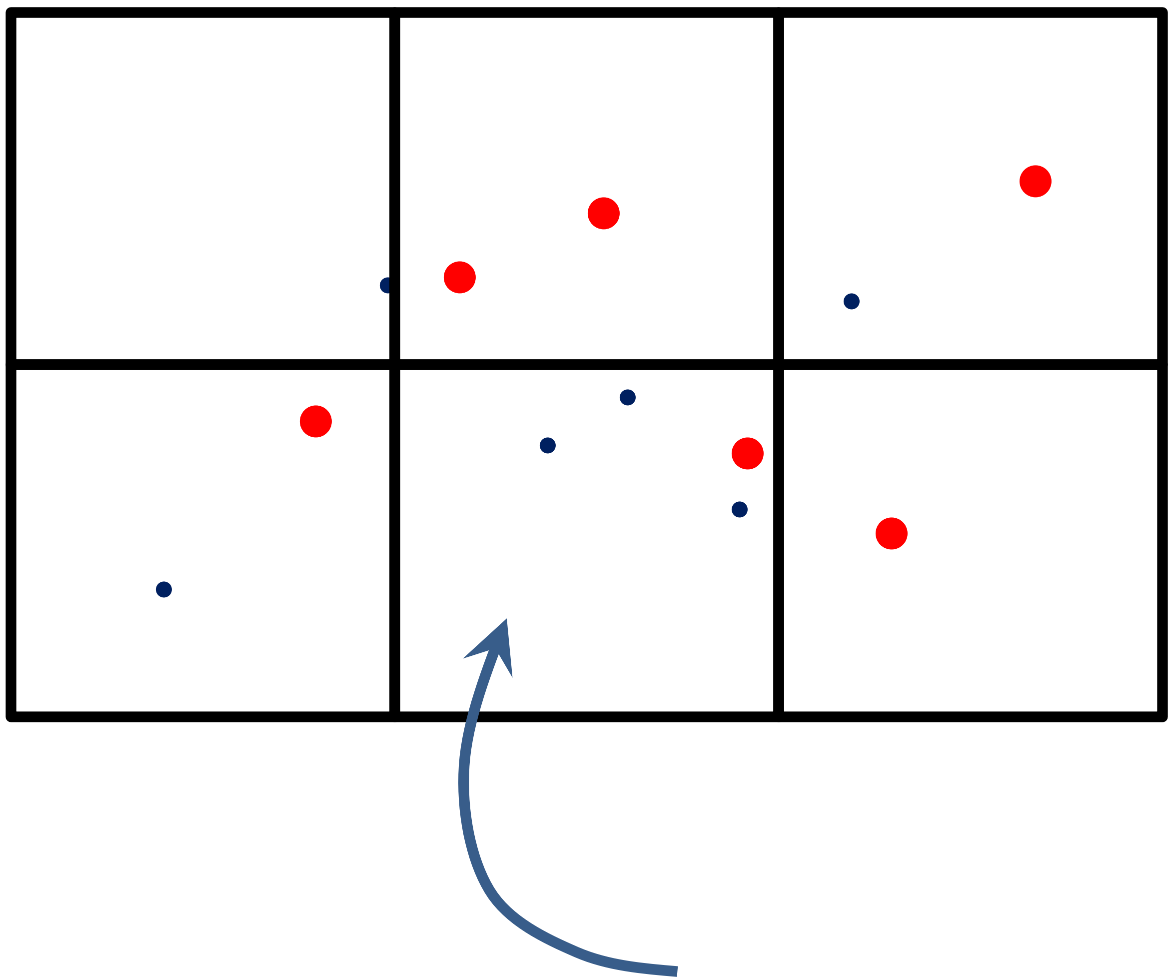
Self-consistent lagrangian methods

Particle-In-Cell method



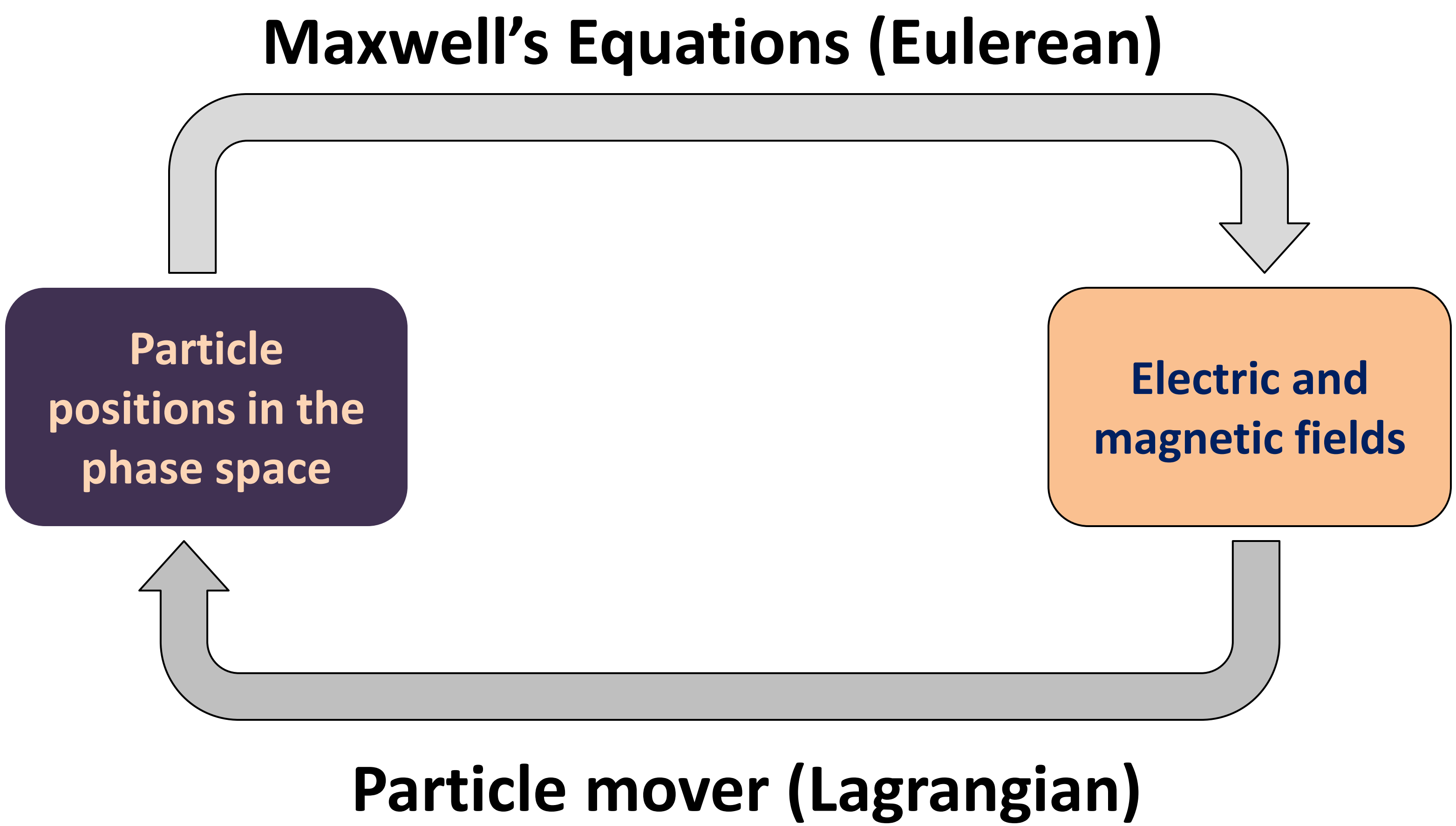
Self-consistent lagrangian methods

Particle-In-Cell method



$\rho_e, \rho_i, \hat{\boldsymbol{T}}_e, \hat{\boldsymbol{T}}_i, \boldsymbol{V}_e, \boldsymbol{V}_i, \boldsymbol{B}, \boldsymbol{E}$

Self-consistent lagrangian methods



Self-consistent lagrangian methods

Limitations of PIC method

- Resolve Debye length
- Resolve plasma frequency

- Resolve Larmor radii and periods
- Unrealistic ion/electron mass ratio

Self-consistent lagrangian methods

Limitations of PIC method

- Resolve Debye length

- Resolve plasma frequency

- Resolve Larmor radii and periods

Gyrokinetic or drift-kinetic PIC

- Unrealistic ion/electron mass ratio

Self-consistent lagrangian methods

Limitations of PIC method

- ~~Resolve Debye length~~
- ~~Resolve plasma frequency~~

Assume quasi-neutrality, solve Maxwell-Faraday and Ampere for fields ???

- ~~Resolve Larmor radii and periods~~
- ~~Unrealistic ion/electron mass ratio~~

Gyrokinetic or drift-kinetic PIC

Self-consistent lagrangian methods

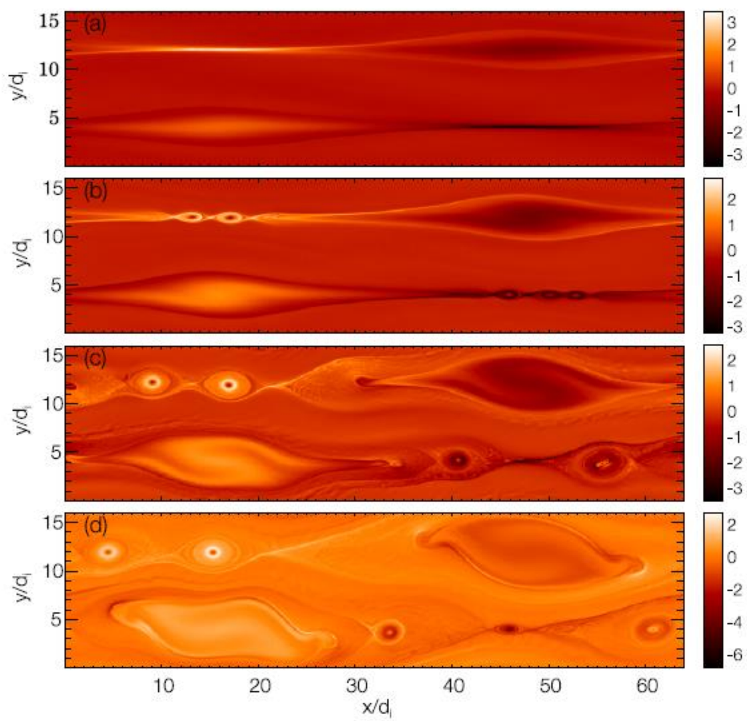


Figure 2. The electron out-of-plane current j_{ez} at four times (t = 11.0, t = 14.0, t = 20.0 and t = 24.0) from a run with B_g = 1.0 and other parameters as in Figure 1 but L_x = 64.0. (a) Note the large island growing on each current layer and the intense current layer driven at each of the x-lines. (b–d) Note the formation, growth and merger of magnetic islands is an ongoing process.

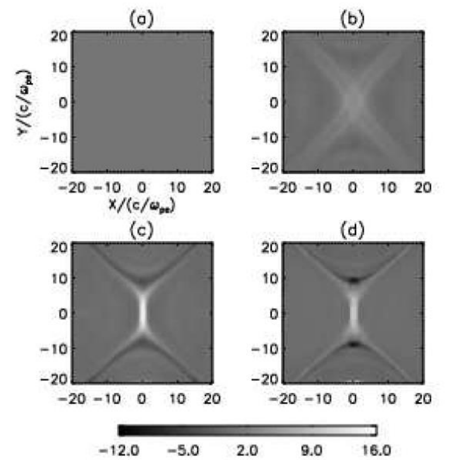


FIG. 4: Time evolution of the spatial distribution of total current density, j_z, in the X-Y plane at (a) $t = 0$, (b) 100, (c) 170 and (d) 250 for $\alpha = 1.20$. The total current density is normalised by the initial value, $j_0 = n_0 e v_{d0}$.

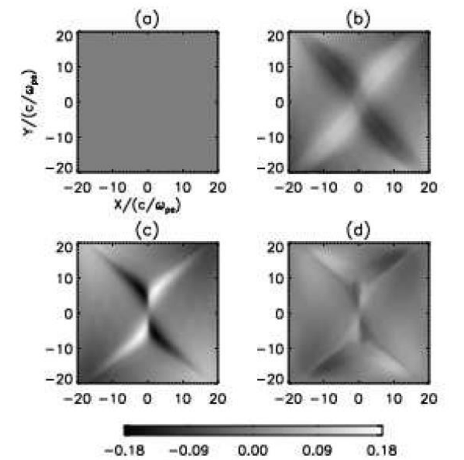


FIG. 5: Time evolution of the spatial distribution of the out-of-plane magnetic field, B_z, at (a) $t = 0$, (b) 100, (c) 170 and (d) 250 for $\alpha = 1.20$. The magnetic field intensity is normalised by the initial value, B_0.

Tsiklauri & Haruki 2007

Drake, Swisdak & co 2005-

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Self-consistent eulerean methods

- Distribution function defined in the phase space ***r***, ***V***

- Solve kinetic equation – conservation equation for the phase space $\frac{\mathrm{d}\, f(\mathbf{r},\mathbf{p},t)}{\mathrm{d}\, t} = 0$

$$\frac{\partial f_e}{\partial t} + \mathbf{v}_e \cdot \nabla f_e - e\left(\mathbf{E} + \frac{\mathbf{v_e}}{c} \times \mathbf{B}\right) \cdot \frac{\partial f_e}{\partial \mathbf{p}} = 0$$

$$\frac{\partial f_i}{\partial t} + \mathbf{v}_i \cdot \nabla f_i + Z_i e\left(\mathbf{E} + \frac{\mathbf{v_i}}{c} \times \mathbf{B}\right) \cdot \frac{\partial f_i}{\partial \mathbf{p}} = 0$$

$$\nabla \times \mathbf{B} = \frac{4\pi \mathbf{j}}{c} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{E} = 4\pi\rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\rho = e\int (Z_i f_i - f_e) d^3p, \quad \mathbf{j} = e\int (Z_i f_i \mathbf{v}_i - f_e \mathbf{v}_e) d^3p,$$

Eulerean methods

Low-dimensional Vlasov-Maxwell methods

- Use of GCA (gyro-kinetic or drift-kinetic approaches) makes it possible to get rid of one V dimension

- 1D3V and 1D2V models: beam propagation in magnetised plasmas

- Assumptions about particle distribution in respect of velocity, “Reduced kinetics” (Gordovskyy & Browning 2016 arxiv)

Eulerean methods

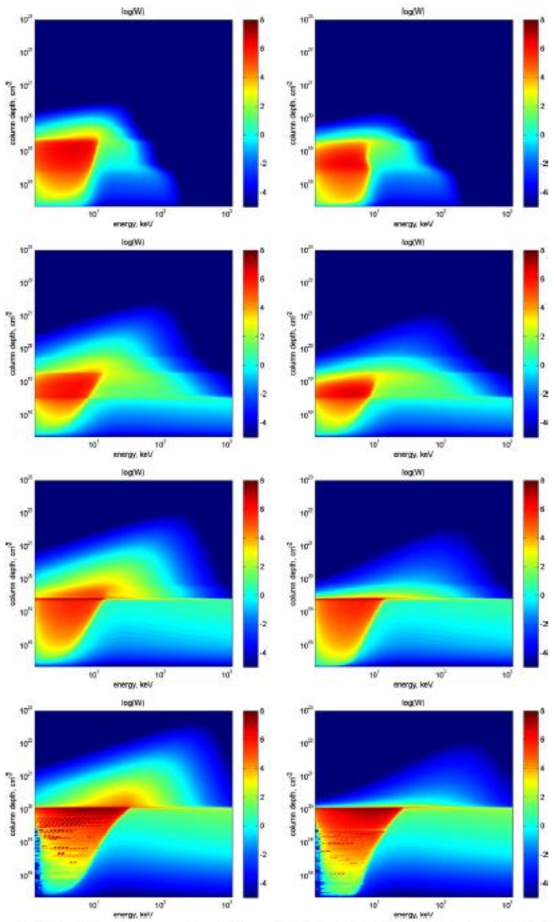

Figure 5. Three-dimensional density of Langmuir wave energy (erg cm^{-4} s) vs. column depth simulated without (left column) and with (right column) a self-induced electric field for the beams with: $\delta = 7$ and $F_0 = 10^{10}$ erg $\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$ (first row); $\delta = 3$ and $F_0 = 10^{10}$ erg $\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$ (second row); $\delta = 3$ and $F_0 = 10^{11}$ erg $\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$ (third row); $\delta = 3$ and $F_0 = 10^{12}$ erg $\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$ (fourth row).

(A color version of this figure is available in the online journal.)

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Zharkova & Siversky 2011

Eulerean methods

Getting rid of evolving fields does not make the problem much simpler.

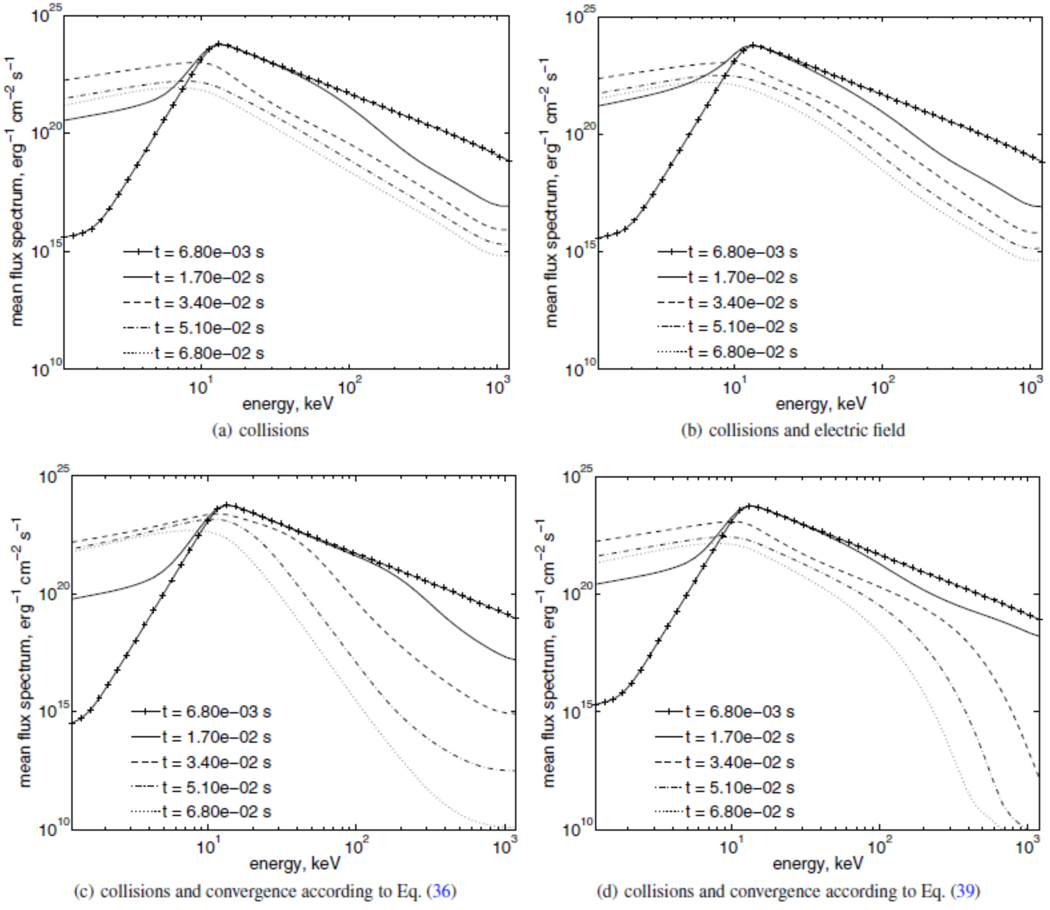

Fig. 11. Mean flux spectra of the electrons injected as a short impulse. The beam parameters are the same as in Fig. 10.

Siversky & Zharkova 2009

Hybrid methods

- Aka “fluid-kinetic approaches”
- Only non-thermal part of plasma is treated using one of the kinetic approaches, the rest is treated as a fluid
- Fluid electrons + kinetic ions
- Fluid plasma + small fraction of kinetic particles
- Particles are kinetic only in a part of the computational domain
- Etc etc