# Transport of Charged Particles in Solar Flares

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On-line talk

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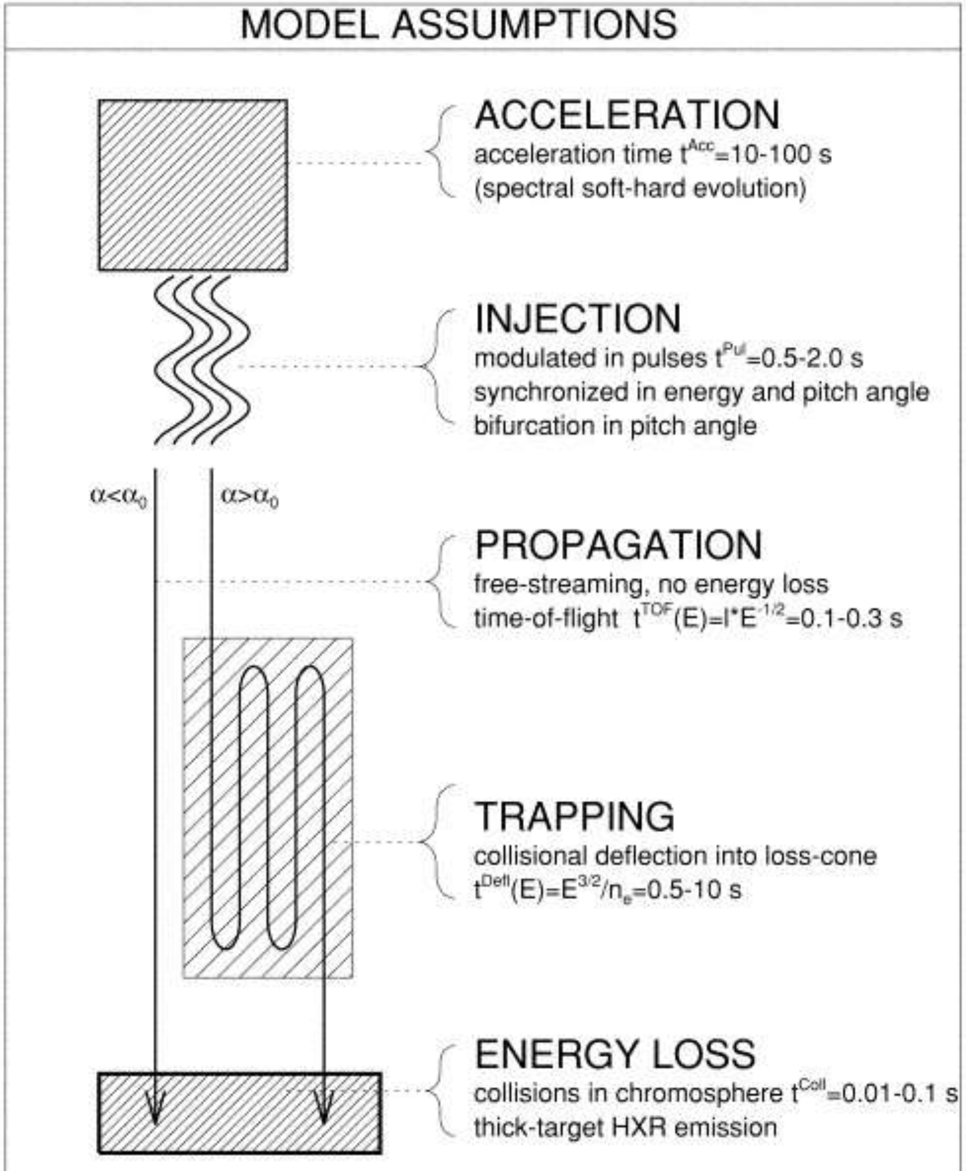
# Plan of the Talk

- Overview
- Regimes of particle transport
- What can microwave data tell us about particle transport: some examples

# TPP/DP Model




Aschwanden 1998

Let us consider a question of how a test particle with a given charge, mass, and velocity propagates through a source with a **linear scale *L***. The answer depends essentially on the amount and sort of the test particles participating in the motion and on the background source properties.

Apparently, the simplest case of the particle transport is a free streaming, when no external force affects the particle motion noticeably.

A number of other regimes will appear if there are external forces – regular or/and random – acting on the particles: gyration, bouncing, drifts, diffusion, advection, and turbulent diffusion.

# Free-Streaming and Time of Flight

- When a particle with a velocity ***v*** propagates freely through a volume with linear scale *L*, the time needed for the particle to cross the entire source is called the **time of flight**, $\tau_{\mathrm{tof}}$ , which is apparently equal to

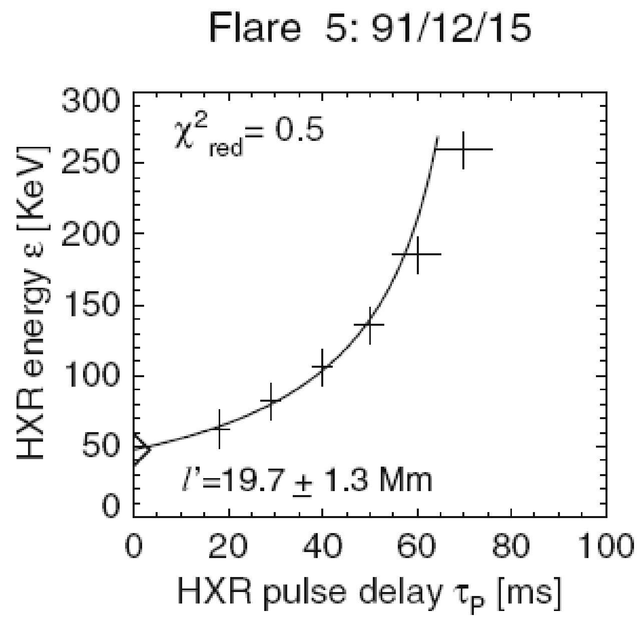
$$\tau_{\mathrm{tof}} = L/v \qquad L_{\mathrm{tof}} = v\tau$$

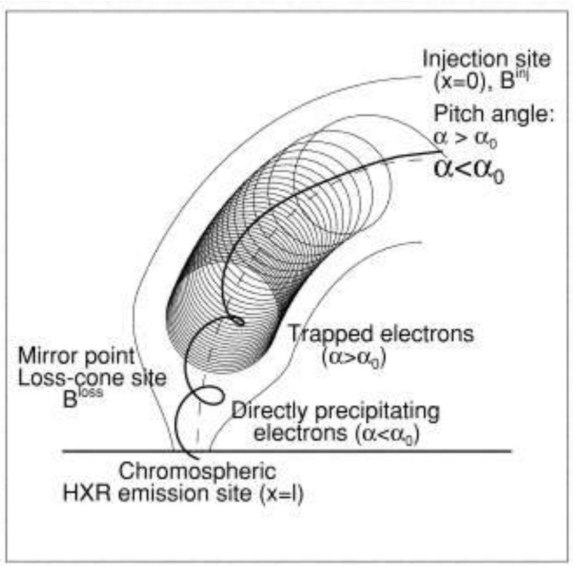
- In fact, **the ability of the particles to stream freely is strongly bounded by the presence of magnetic field**

$$r_L = cp_\perp/(e_i B)$$

- ~ $3\times10^6$ ($\beta\gamma/B$) [cm] for protons
- ~ $1.7\times10^3$ ($\beta\gamma/B$) [cm] for electrons

**Conclusion 1: particle transport in the presence of magnetic field can be highly anisotropic**



# Guiding Center, Drifts, and Adiabatic Invariants

$$\overline{\boldsymbol{p}}'_\perp = \frac{1}{T}\int_{\tau_0}^{\tau_0+T} \boldsymbol{p}'_\perp(\tau)\mathrm{d}\tau = 0, \qquad \overline{p}'_{\|} = p'_{\|0} \qquad T = 2\pi/\omega'_c$$

$$\boxed{\boldsymbol{v}_c = \boldsymbol{v}_E + \boldsymbol{v}_{\|}} \qquad \boxed{\boldsymbol{v}_E = \frac{c}{B^2}\boldsymbol{E}\times\boldsymbol{B}}$$

$$\boldsymbol{E}_{\text{eff}} = \boldsymbol{\mathcal{F}}/e \qquad \boldsymbol{v}_d = \frac{c}{eB^2}\boldsymbol{\mathcal{F}}\times\boldsymbol{B}$$

$$\boldsymbol{v}_c = \dot{\boldsymbol{r}}_c = v_{\|}\boldsymbol{e}_{\|} + \frac{c}{B^2}\boldsymbol{E}\times\boldsymbol{B} + \frac{mc}{eB^2}\boldsymbol{g}\times\boldsymbol{B} + v_{\|}r_{g\|}\boldsymbol{e}_{\|}\times(\boldsymbol{e}_{\|}\cdot\nabla)\boldsymbol{e}_{\|} + \frac{v_\perp r_{g\perp}}{2B}\boldsymbol{e}_{\|}\times\nabla B$$

$$\frac{p_\perp^2}{B} = \text{const}$$

**Conclusion 2: particles are being mirrored from regions with strong magnetic field**

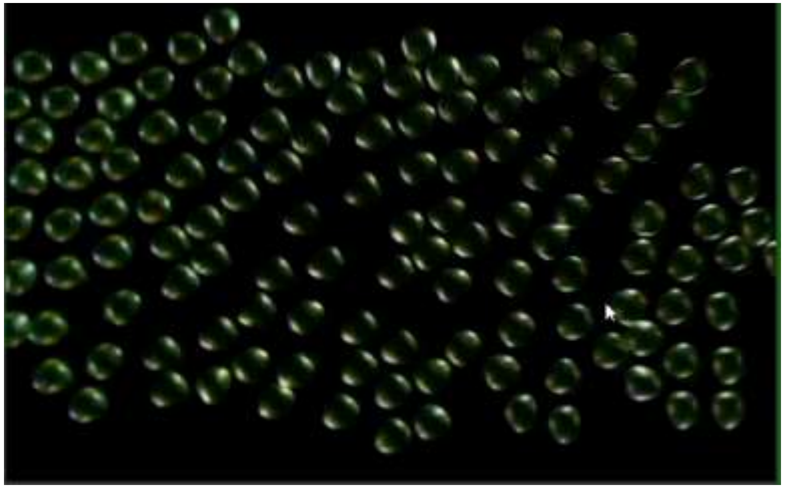
# Scattering & Spatial Diffusion

Fick's law in an isotropic gas

$$\boldsymbol{i}(\boldsymbol{r},t) = -\kappa \nabla n(\boldsymbol{r},t)$$

$$\kappa = \frac{1}{3} v \Lambda$$

$\boxed{\Lambda \text{ is the mfp}}$



**Modification in a magnetized plasma**

$$\cancel{n_r m_r \frac{\partial \boldsymbol{u}_r}{\partial t}} = -\nabla P_r + \frac{q n_r}{c} \boldsymbol{u}_r \times \boldsymbol{B} - n_r m_r \nu_r \boldsymbol{u}_r$$

$$\boldsymbol{i}_r = n_r \boldsymbol{u}_r \qquad \boxed{\boldsymbol{i}_r = -\hat{\kappa}' \nabla n_r + \boldsymbol{g} \times \nabla n_r} \qquad \boldsymbol{g} = \frac{\kappa_{||} \omega_{Br} \tau_r}{1 + (\omega_{Br} \tau_r)^2} \frac{\boldsymbol{B}}{B}$$

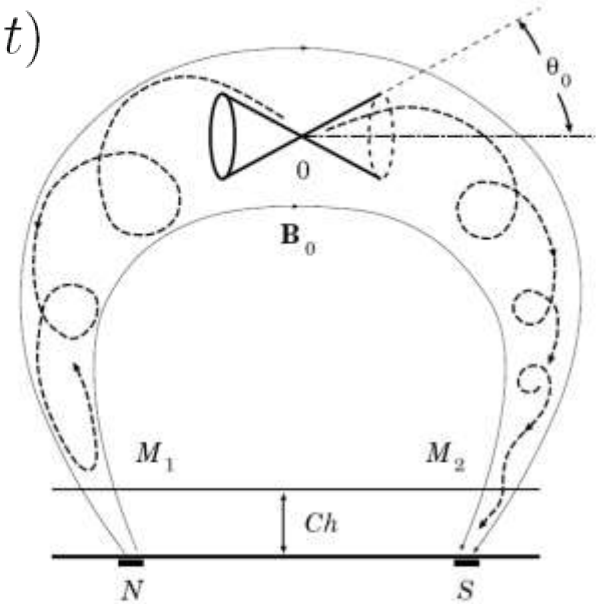
$$\hat{\kappa}' = \begin{pmatrix} \kappa_\perp & 0 & 0 \\ 0 & \kappa_\perp & 0 \\ 0 & 0 & \kappa_{||} \end{pmatrix} \qquad \kappa_\perp = \frac{\kappa_{||}}{1 + (\omega_{Br} \tau_r)^2} \qquad \omega_{Br} = \frac{qB}{m_r c}$$

**Conclusion 1': particle diffusion in the presence of magnetic field can be highly anisotropic**

# Particle Transport in a Thin Loop

$$\frac{\partial f}{\partial t} = -c\beta\cos\vartheta\frac{\partial f}{\partial s} - \frac{c\beta\sin\vartheta}{2}\frac{\mathrm{d}\ln B}{\mathrm{d}s}\frac{\partial f}{\partial\vartheta} - \frac{\partial}{\partial E}(\dot{E}_L f) + \frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\sin\vartheta D_{\vartheta\vartheta}\frac{\partial f}{\partial\vartheta}$$

$$+ \frac{1}{p^2(E)}\frac{\partial}{\partial E}(p^2(E)D_{EE})\frac{\partial f}{\partial E} + \cdots + S(E,\vartheta,s,t)$$




**Most important parameters:**

isotropization time $\boxed{\tau_s \sim 1/\langle D_{\vartheta\vartheta}\rangle}$

mfp $\Lambda = \beta c/\langle D_{\vartheta\vartheta}\rangle$

diffusion coefficient $\kappa = \beta c\Lambda/3$

escape time $\boxed{\tau_e(E) = L^2/\kappa = \frac{3L^2}{\beta^2 c^2}\langle D_{\vartheta\vartheta}\rangle}$

# Collisional Scattering

$$\sigma_{ri} \approx \pi \left( \frac{qe}{m_{ri} v_{ri}^2} \right)^2 \ln \Lambda_{\rm C} \quad \Lambda_{ri} \approx \frac{1}{n_i \sigma_{ri}} \quad \kappa_{\|} = \frac{1}{3} v_{Tr} \Lambda_r \propto T^{5/2}.$$

$$D_{\vartheta\vartheta(\mathrm{C})} = \frac{c}{\lambda_0 \beta^3 \gamma^2} \qquad \lambda_0 = 10^{24}/(n(s) \ln \Lambda_{\rm C})$$

# Resonant Scattering by Waves

$$W_w(k) = \frac{q-1}{k_0} \left( \frac{k_0}{k} \right)^q W_{\rm tot}; \qquad W_{\rm tot} = \int_{k_0}^{\infty} W_w(k) \mathrm{d}k \qquad \omega - kc\beta \cos\theta \cos\vartheta - \frac{\hat{s}\omega_{\rm Be}}{\gamma} = 0$$

$$\begin{pmatrix} D^w_{\vartheta\vartheta} \\ D^w_{\rm pp} \end{pmatrix} = \omega_{\rm Be} \sin^2\vartheta |\cos\vartheta|^{q-1} \left( \frac{k_{th} c}{\omega_{\rm Be}} \right)^{q-1} \frac{(\beta\gamma)^{q-1}}{\gamma} \left( \frac{8\pi W_{\rm tot}}{B^2} \right) \left[ \begin{matrix} I^w_{\vartheta\vartheta}(q) \\ \frac{m_p^2 c^2 \beta_{\rm A}^4 I^w_{\rm pp}}{\beta^2} \end{matrix} \right]$$

**Conclusion 3: different scattering regimes are distinguishable by their dependence on the particle energy**

# Transport Transvers to Magnetic Field

The transverse displacement of the particle as it crosses one correlation cells is

$$l_\perp \approx \widetilde{B} L_c / B_0$$

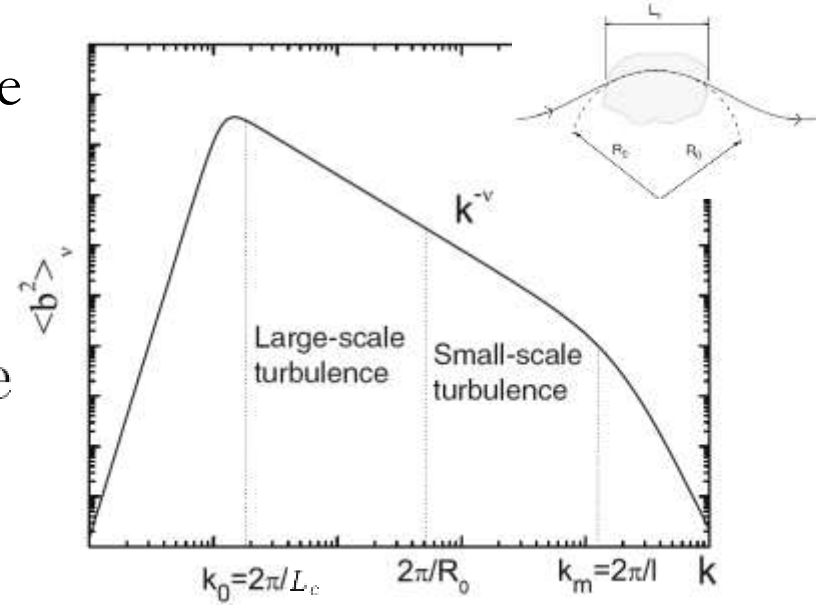
During a given time $t$ the particle makes $N = |v_{\|}| t / L_c$ steps

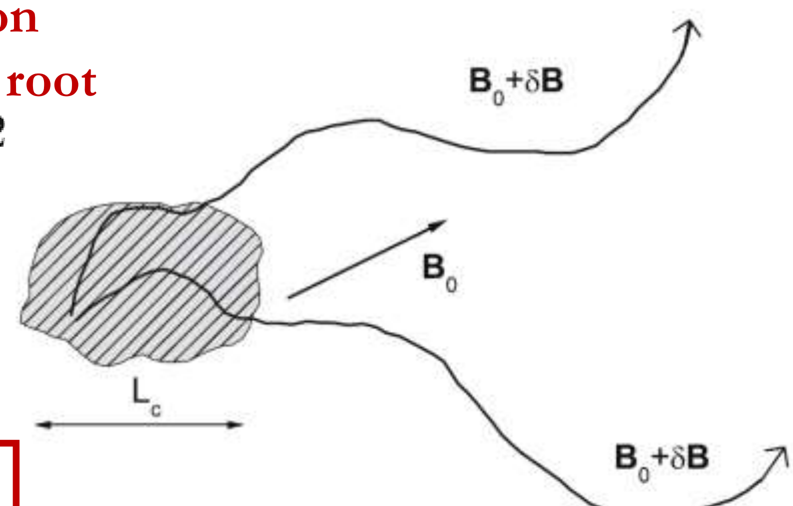
**If the walk is truly random then the displacement $\Delta L_\perp$ in the transverse direction over this time *t* is proportional to the square root of the number of steps** $\Delta L_\perp \approx l_\perp N^{1/2}$

$$(\Delta L_\perp)^2 \approx (\langle \widetilde{B}^2 \rangle / B_0^2) L_c |v_{\|}| t$$

$$\boxed{\kappa_\perp \approx (\langle \widetilde{B}^2 \rangle / B_0^2) L_c |v_{\|}|}$$

**Conclusion 4: transverse diffusion can be greatly enhanced by turbulent magnetic field**




adopt that $\Lambda_{\|} \gg L_c$

# Advection and Turbulent Diffusion

If the plasma moves, the test particles are being picked up and, besides the diffusion, take part in the advection transfer with the fluid velocity $\boldsymbol{u}(\boldsymbol{r}, t)$. The full flux produced by the admixture particles is, therefore, composed of **diffusive** and **advective** terms: $i_\alpha(\boldsymbol{r},t) = n(\boldsymbol{r},t)u_\alpha(\boldsymbol{r},t) - \kappa_{\alpha\beta}\nabla_\beta n(\boldsymbol{r},t)$

$$-\frac{\mathrm{d}}{\mathrm{d}t}\int_V n\,\mathrm{d}V = \oint_S \boldsymbol{i}\cdot\mathrm{d}\boldsymbol{S}$$

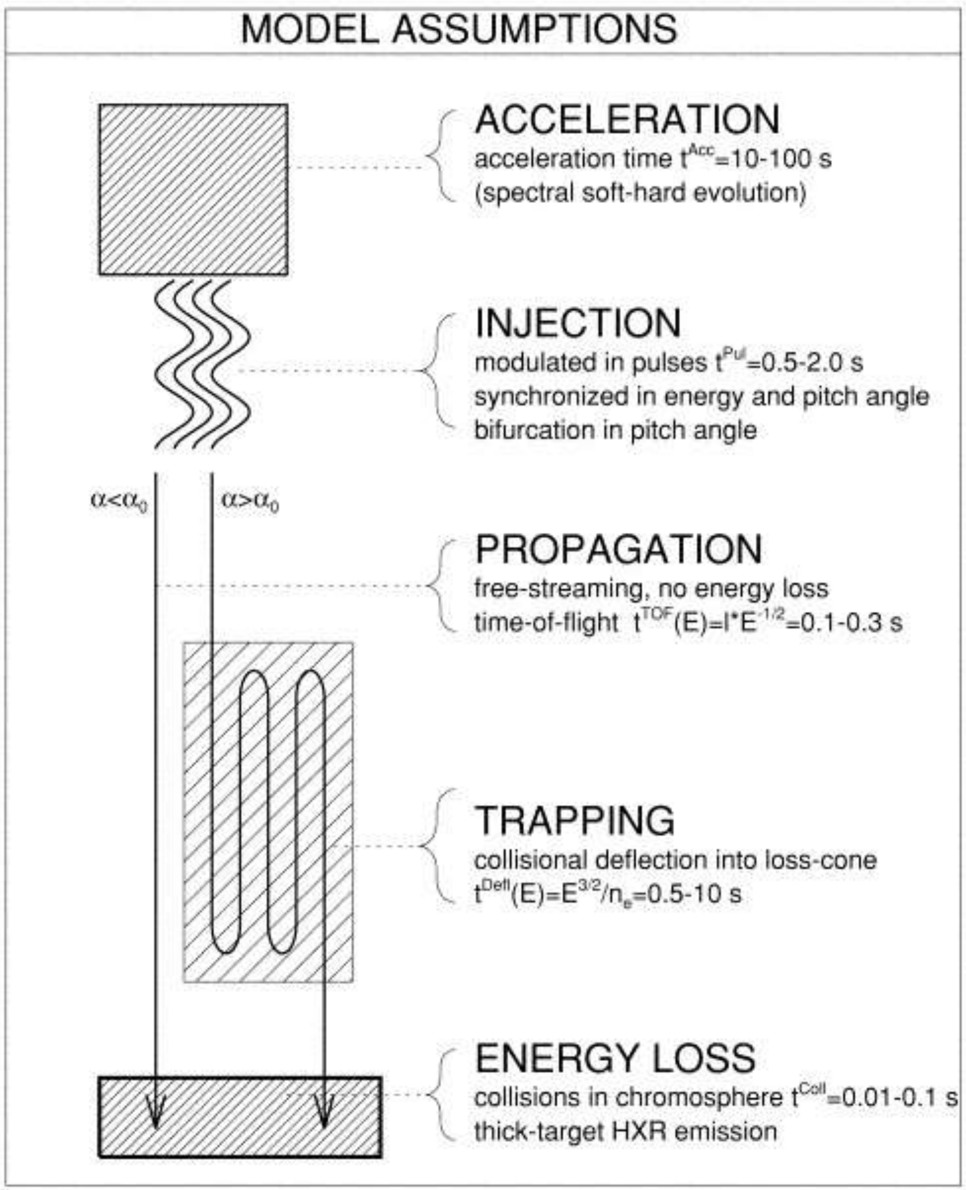
$$\frac{\partial n}{\partial t} + \nabla\cdot n\boldsymbol{u} = \nabla_\alpha \kappa_{\alpha\beta}\nabla_\beta\, n$$

**Péclet number** $\mathrm{Pe} = uL/\kappa$

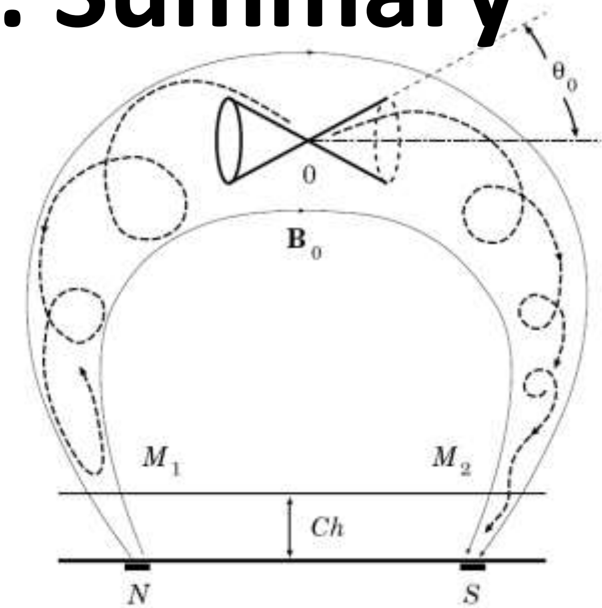
For $\kappa \ll uL$ $\quad \kappa \Rightarrow \chi \approx uL$

**Conclusion 5: turbulent diffusion controls the particle transport if the local mfp is small such as Péclet number is large; thus, the apparent mfp can be larger or even much larger than the local mfp**

# Transport Regimes: Summary




Aschwanden 1998




- Precipitation/escape
- Trapping: magnetic
- Trapping: turbulent
- Diffusion: weak
- Diffusion: moderate
- Diffusion: strong
- Diffusion: turbulent

## GS continuum radio emission can be produced by any of

*(i) a magnetically trapped component* or

***(ii) a precipitating/escaping component***, or

*(iii)* ***the primary component within the acceleration region***.

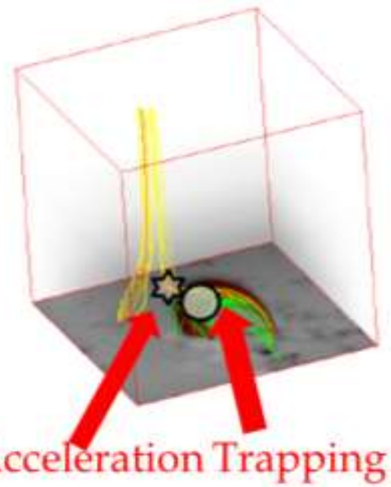
These three populations of fast electrons produce ***radio emission with distinctly different*** characteristics. Indeed,

**(i)** in the case of ***trapping*** the electrons are accumulated at the looptop (Melnikov et al. 2002), and the ***radio light curves must be delayed*** by roughly the trapping time relative to accelerator/X-ray light curves.

**(ii)** In the case of free electron propagation, untrapped precipitating electrons are more evenly distributed in a tenuous loop, and ***no delay*** longer than *L/v* is expected. However, even with a roughly uniform electron distribution, most of the radio emission comes from loop regions with the strongest magnetic field. **Spectral indices of the radio- and X-ray- producing fast electrons differ by 1/2 from each other**.

**(iii)** In the case of **radio emission from the acceleration region**, even though the residence time that fast electrons spend in the acceleration region can be relatively long, the *radio and (thick-target, footpoint) X-ray light curves are proportional to each* ***other*** simply because the flux of the X-ray producing electrons is equivalent to the electron loss rate from the acceleration region.

# Example; 2002-Apr-11

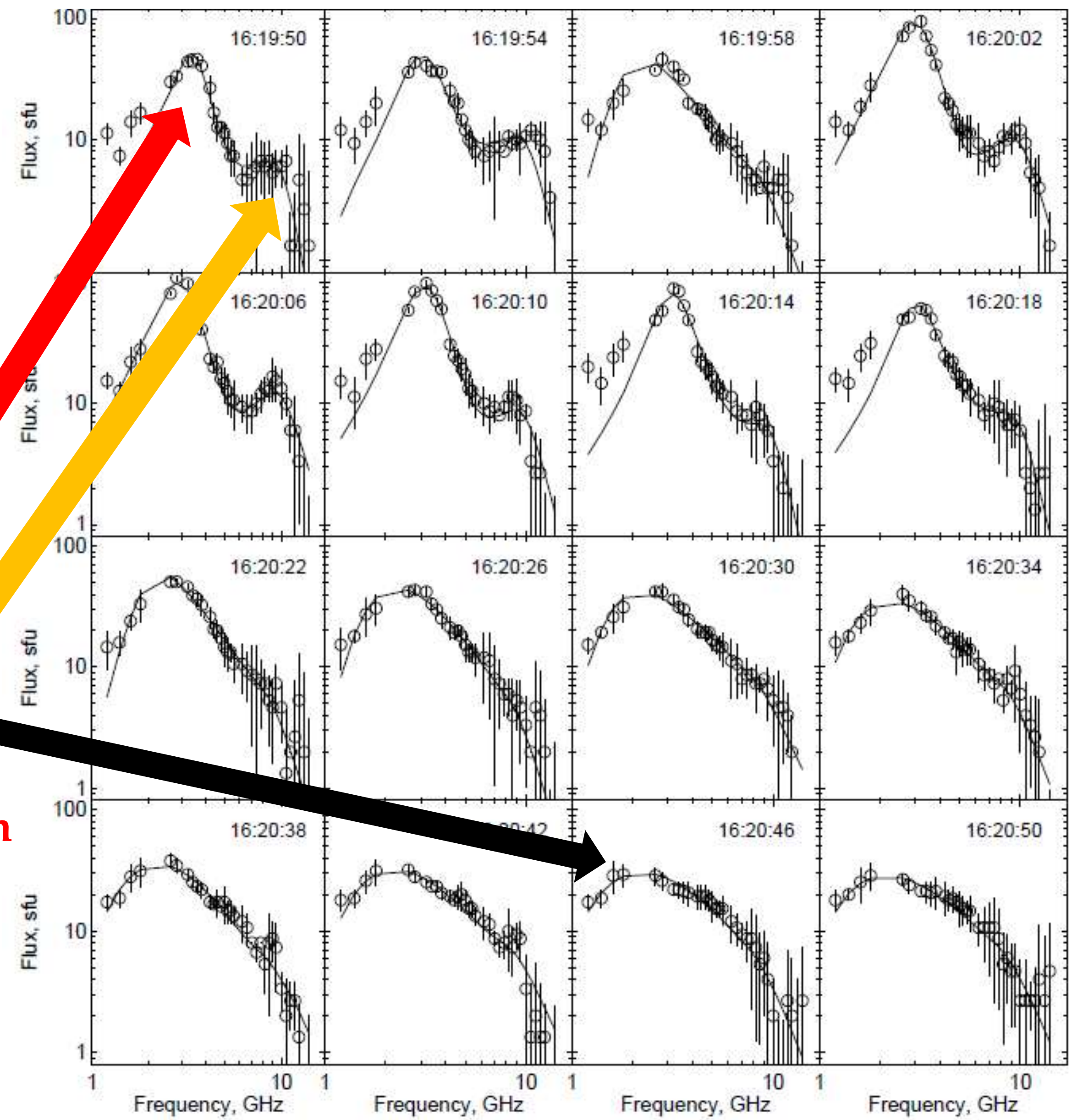



**Emission from:**

Acceleration region

Precipitating electrons

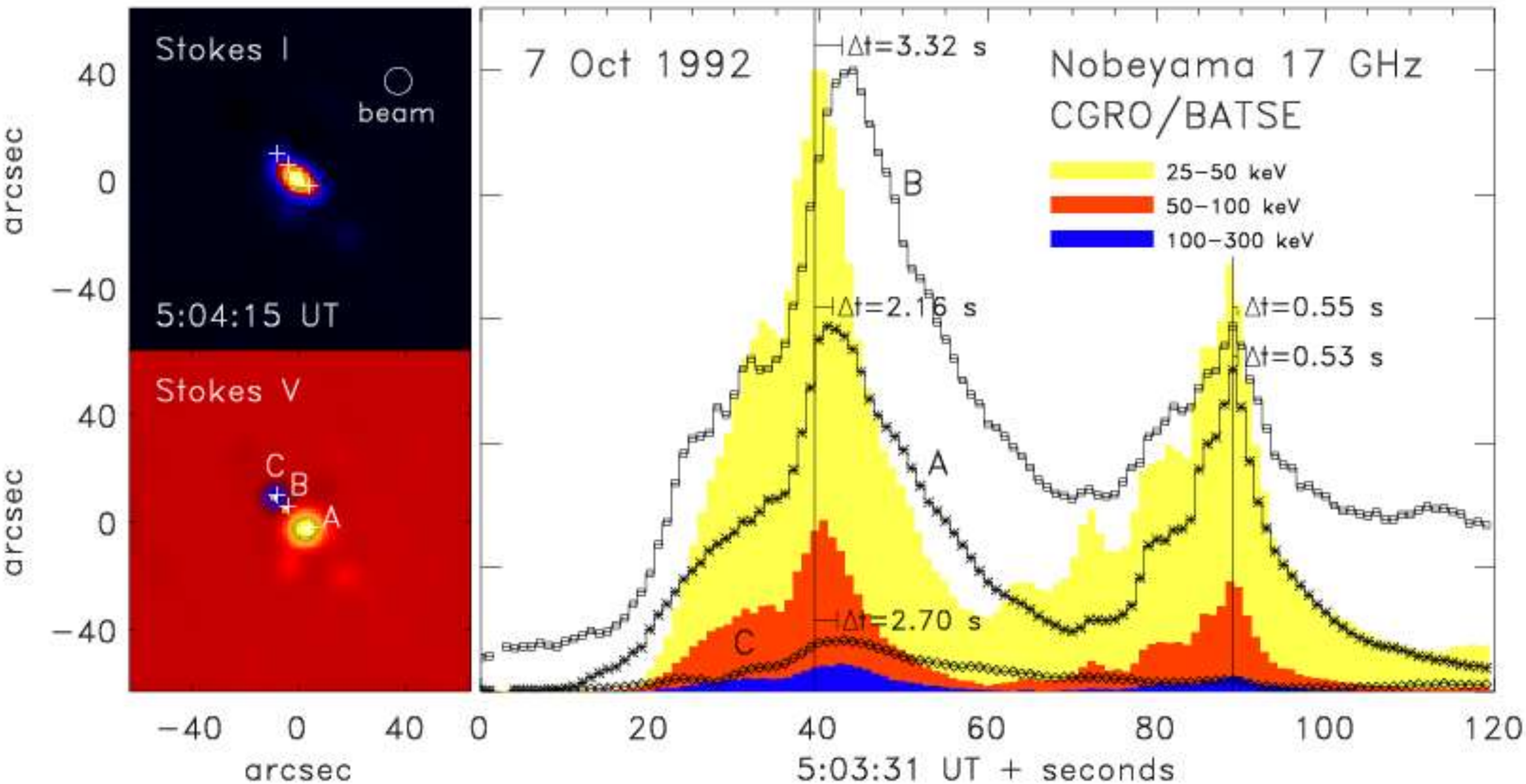
Trapped component




**NB:** microwave emission from the acceleration region can be very narrowband

Fleishman et al. 2013

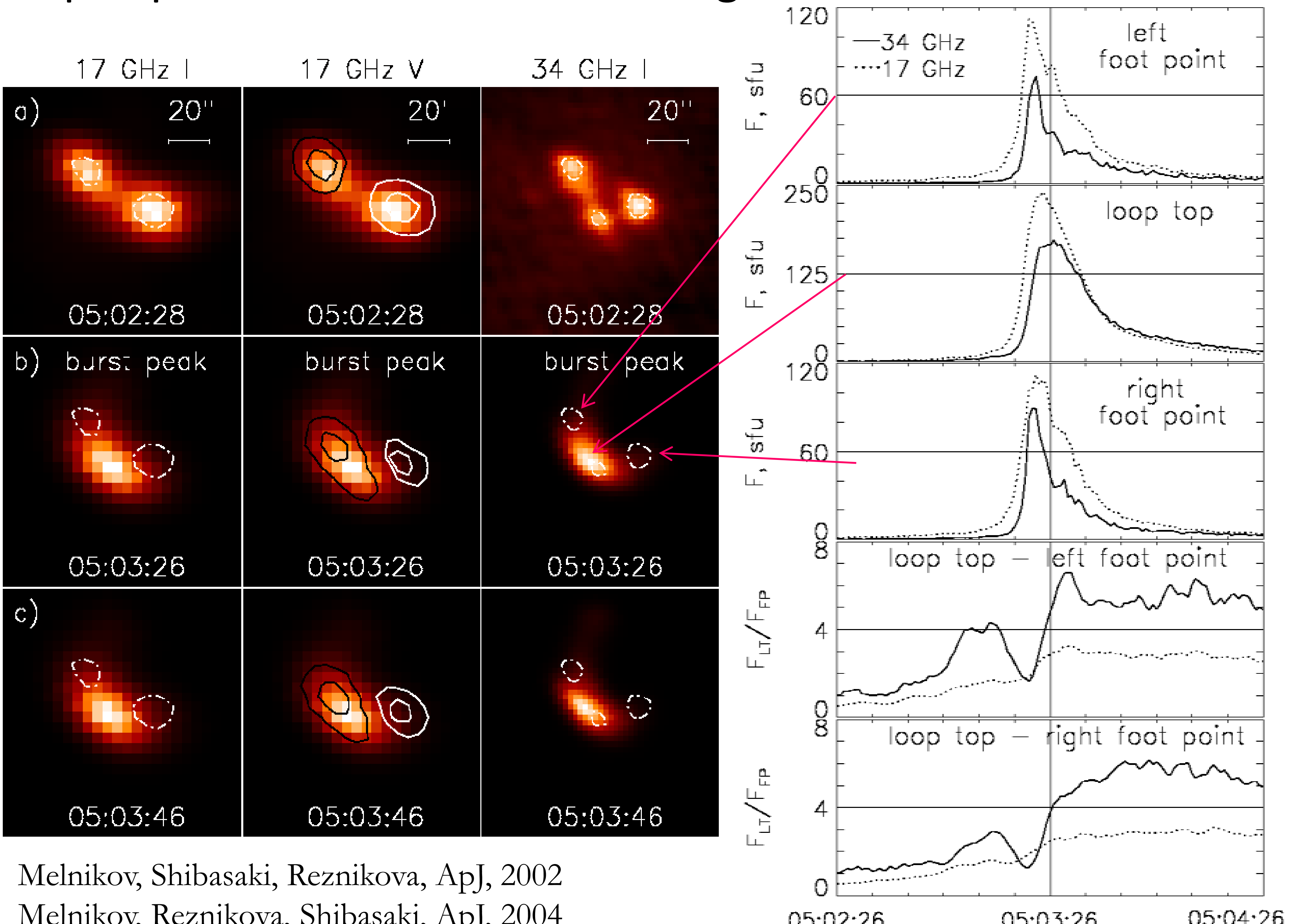
# Time Delays: Various Transport Regimes




Bastian et al 1998

# Loop-top source: weak diffusion regime

*13 March 2000*




Melnikov, Shibasaki, Reznikova, ApJ, 2002

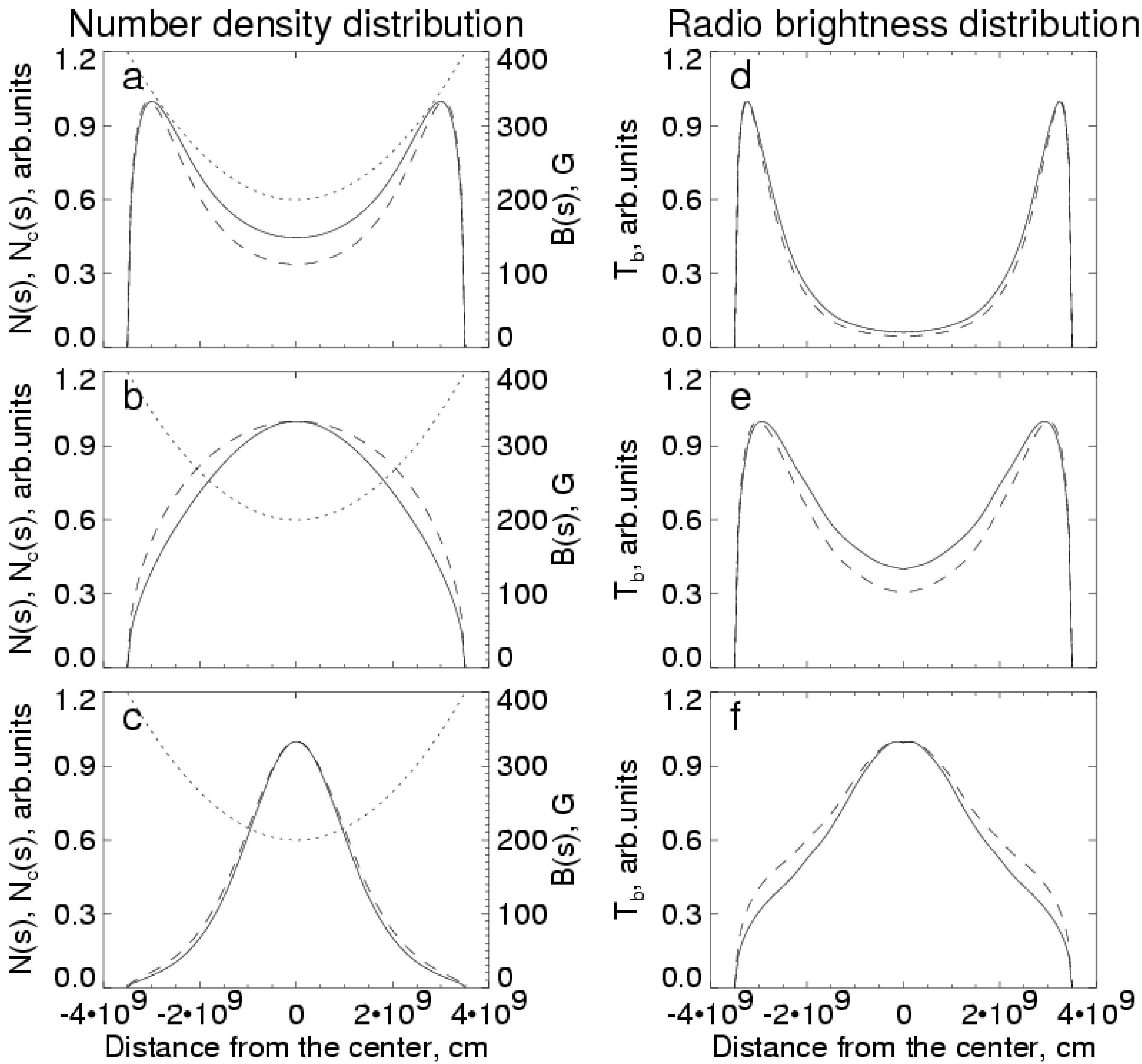
Melnikov, Reznikova, Shibasaki, ApJ, 2004

# Results of simulations

Beam-like injection

Isotropic injection

Pancake injection




Melnikov V.F., Shibasaki K., Reznikova V.E. ApJ, 2002, 580, L185

# Weak Diffusion Regime: Spectral Hardening

Trap properties

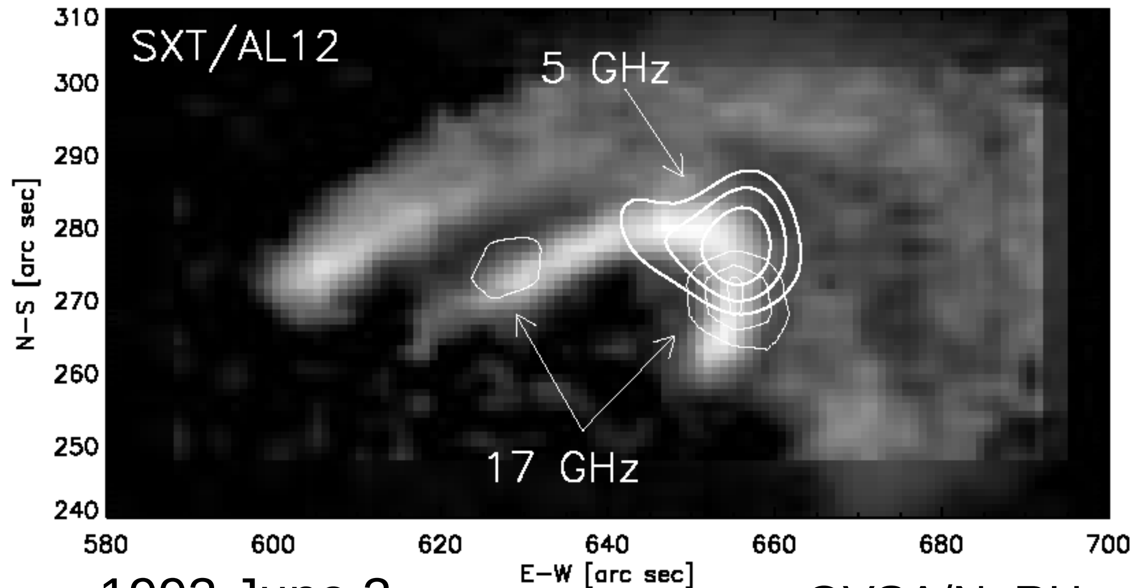
Lee, Gary, & Shibasaki 2000

A comparison of successive flares yielded trap densities of 5 x $10^9$ $cm^{-3}$ in the first, and 8 x $10^{10}$ $cm^{-3}$ in the second.

Anisotropic injection

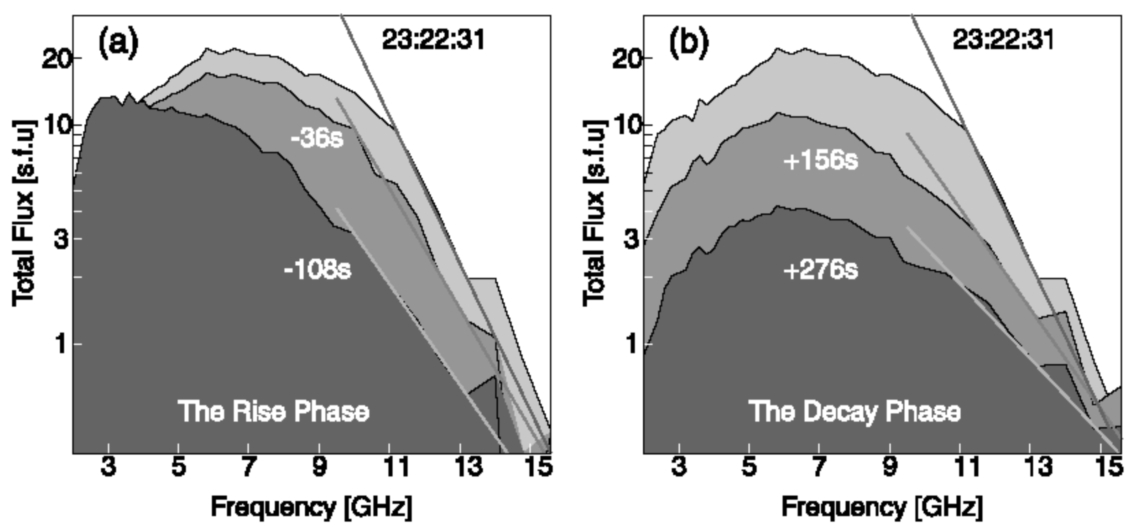
Lee & Gary 2000

Showed that the electron injection in the first flare was best fit by a beamed pitch angle distribution.

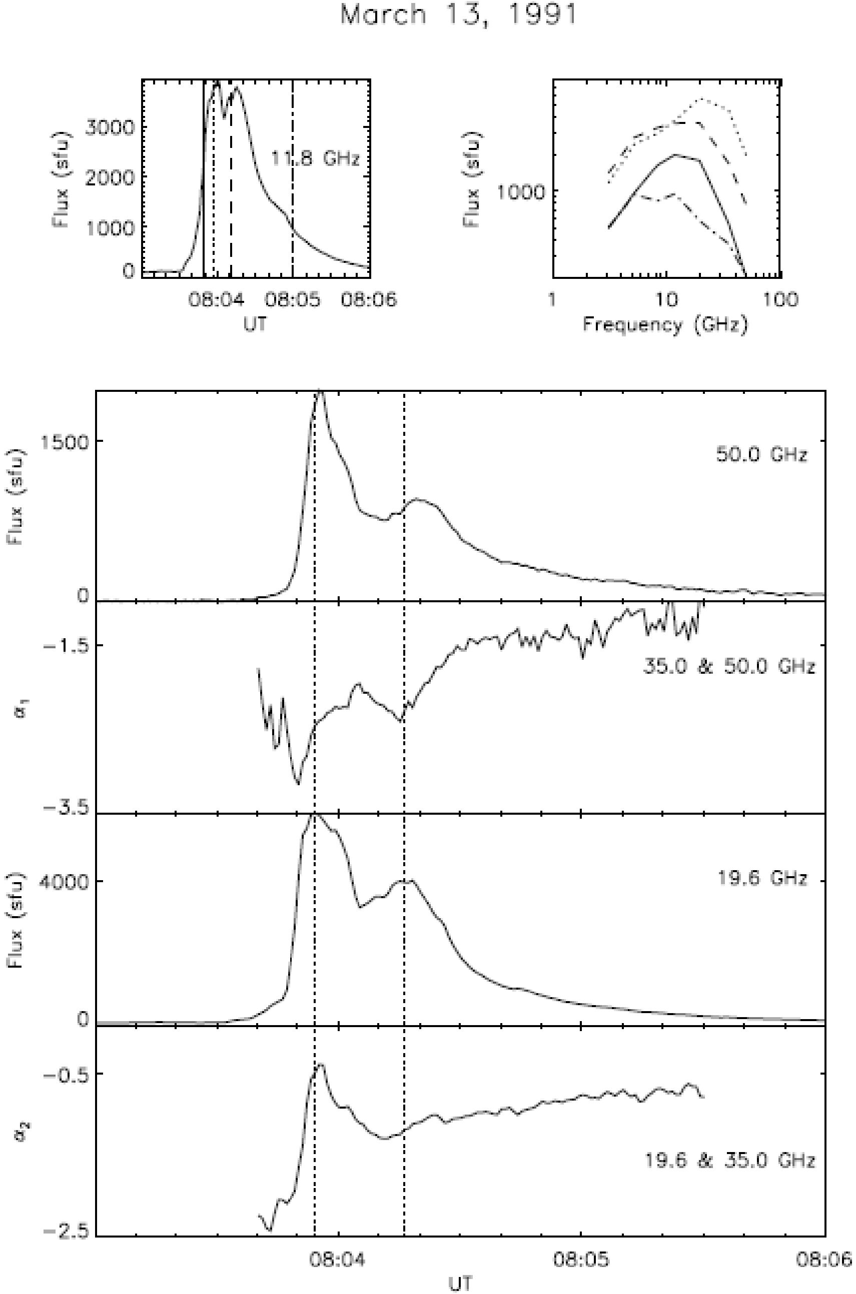


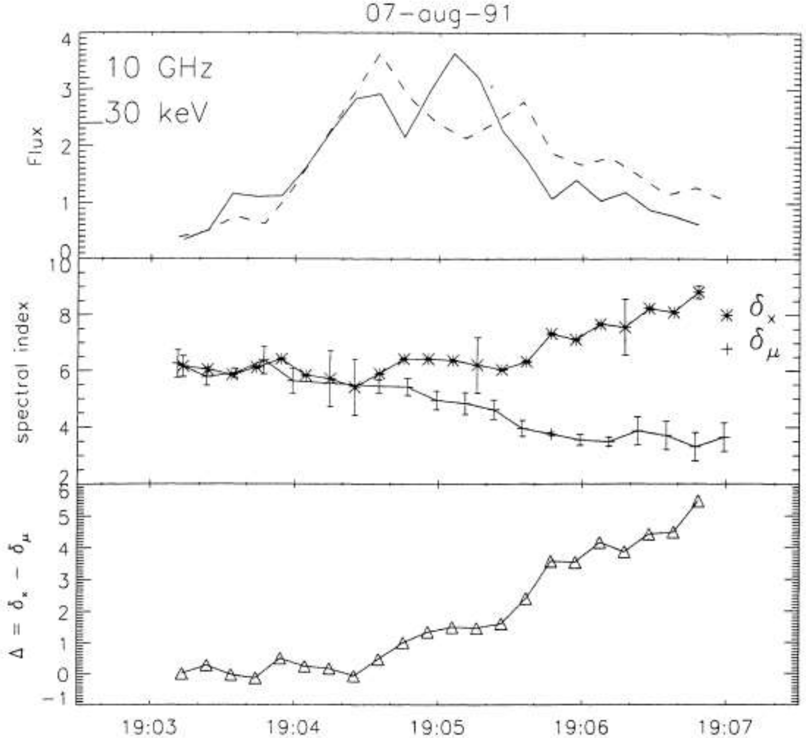

1993 June 3

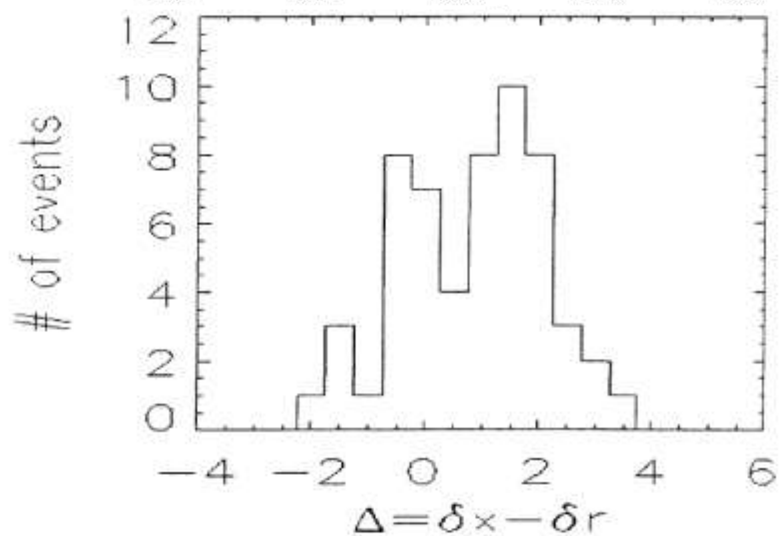
OVSA/NoRH



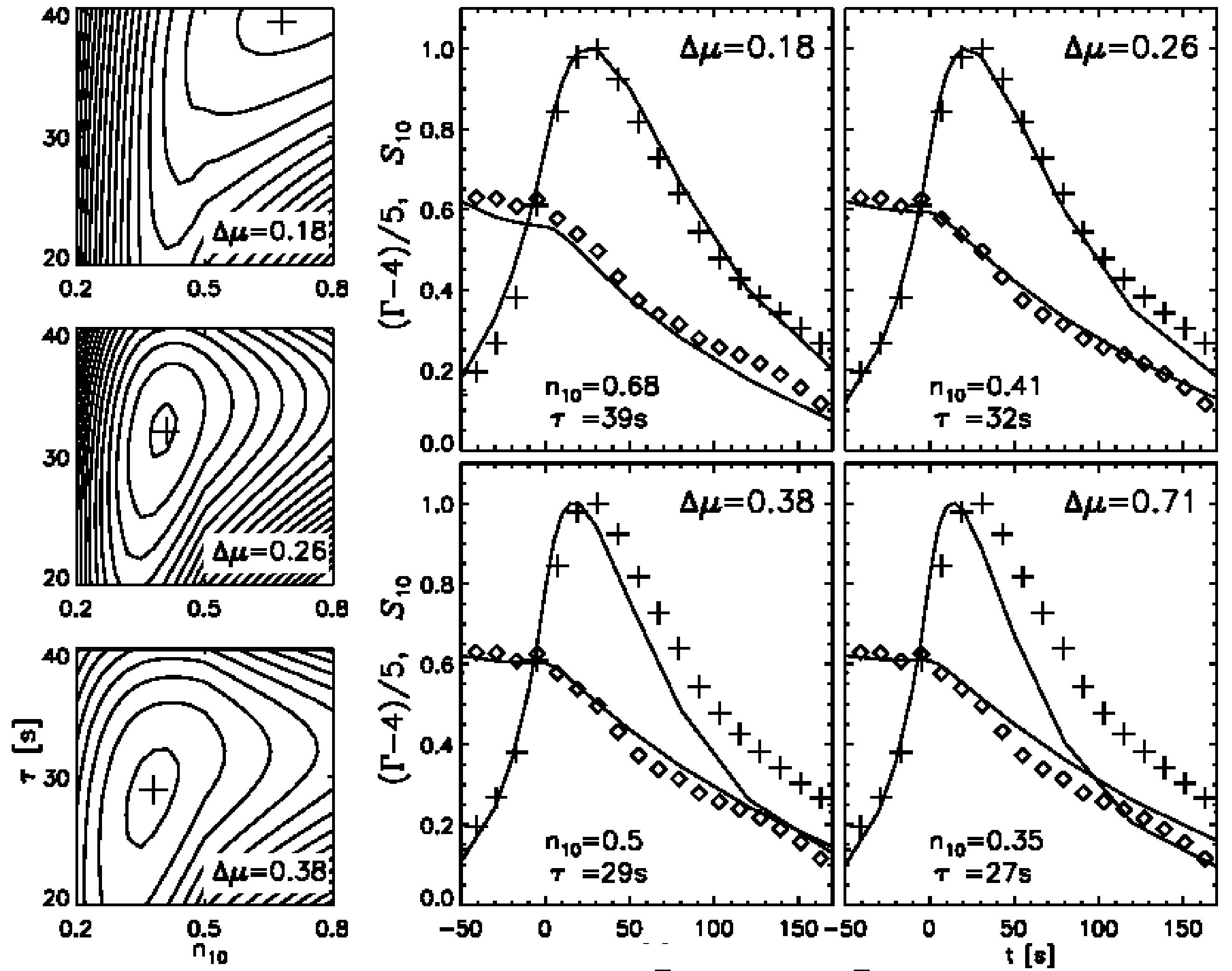
# Weak Diffusion Regime: Spectral Hardening



# Weak Diffusion Regime: Anisotropic Injection and Isotropization




$\mu_o = 0$

OVSA

1993 June 3

$$P_\gamma(\mu)|_{s=0} = \frac{1}{\sqrt{\pi}\Delta\mu} \exp\left[-4\,\frac{(\mu-\mu_0)^2}{\Delta\mu^2}\right].$$

Lee & Gary 2000

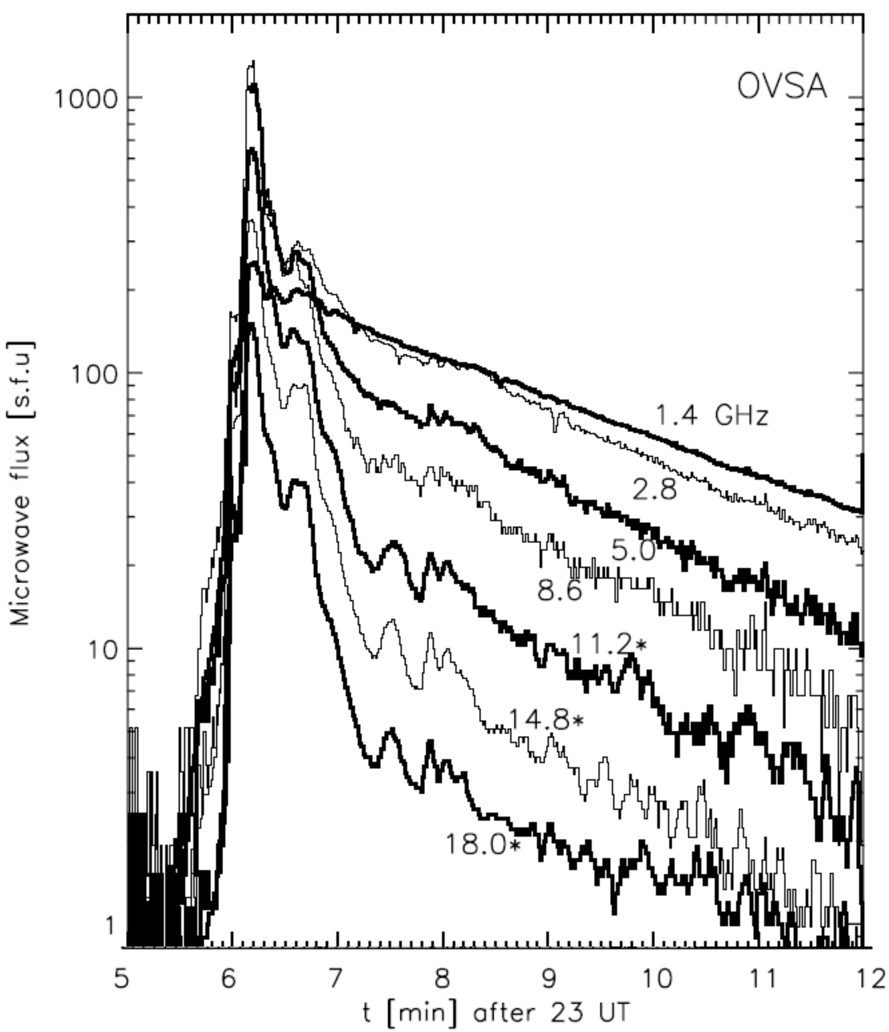
# Strong Diffusion Regime: NO Spectral Hardening

In the weak diffusion regime the low-energy electrons are being scattered and precipitate into the loss-cone faster than the high-energy electrons, which manifests itself in significant hardening of the MW spectrum. There are many cases, where this does not occur; thus, an enhanced scattering with a different energy dependence of the isotropization time must be involved: either strong or moderate diffusion regime.

**Moderate diffusion regime** for **$B_{fp}$ = 800 G, L = 1.4x$10^{10}$cm** implies:

- **Mirror ratio: ~ 180**
- Loss-cone angle ~ 4.3$^{o}$
- **Loop-top $B_{lt}$ ~ 5 G**

**Strong diffusion regime will be consistent with the data if the spectral index of the turbulence is about 2** (see also Bastian et al. 2007, Fleishman et al. 2016 and many others)




Lee & Gary 2002

# Keep Transported and Accelerated!